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A METHOD FOR COMPUTING THE PRESSURE DISTRIBUTION IN THE ELASTIC MODE OF SINGLE-WELL FORMATION DEVELOPMENT

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A B S T R A C T

The process of developing a single-well oil reservoir in an elastic mode is considered, described by the model of a nonstationary plane-radial filtration flow of a single-phase liquid in a porous medium. For the proposed model, the laws of change in the time of the well flow rate and the pressure at the bottom of the well are considered set. And the pressure distribution in the formation at the initial moment of time, as well as the pressure at the outer boundary of the formation, are considered unknown. The task of determining the dynamics of pressure distribution in the reservoir based on the proposed model is set. This problem belongs to the class of boundary inverse problems without initial conditions. First, a discrete analogue of the inverse problem is constructed using the method of difference approximation. A computational algorithm is proposed for the numerical solution of the resulting system of linear algebraic equations up to a predetermined time layer l. In this case, the pressure at the nodal points of a predetermined triangular region is not determined. And for solving a system of linear equations, starting from the time layer $l+1$, a special representation is proposed. At the same time, at each time layer, the system of linear equations is split into two mutually independent linear subsystems, each of which is solved independently, independently of each other. As a result of splitting, an explicit formula for determining the approximate pressure value at the outer boundary of the formation and a recurrent formula for determining the pressure distribution in the formation, starting with $l+1$ time layer, were obtained. Based on the proposed computational algorithm, numerical experiments were carried out for a model single-well oil reservoir.

Keywords: elastic development mode; single-phase filtration; plane-radial fluid flow; boundary inverse problem without initial conditions; difference approximation method.

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Introduction

It is known that the development of oil reservoirs in the initial period is carried out in a natural elastic mode. The condition for the manifestation of the elastic regime is the excess of reservoir pressure over the pressure of saturation of oil with gas. In the elastic mode of development, oil is in a single-phase state and oil flow to wells occurs due to the use of energy from the elastic expansion of the oil itself and the formation rock [1-4]. It should be noted that reservoir pressure, which determines the energy capabilities of a reservoir, is one of the main characteristics of an oil reservoir and affects almost all indicators of reservoir development, including well productivity and a change in the development regime [4-7]. Therefore, in the process of developing an oil reservoir in an elastic mode, constant monitoring of the pressure distribution in the reservoir is required. In the practice of oil field development, the dynamics of reservoir pressure is mainly determined by measuring it in specially stopped wells, which are associated with certain difficulties and significant losses in oil production. In this regard, in order to control the pressure distribution in the elastic mode of formation development, it becomes necessary to use a mathematical model of the non-stationary filtration flow of a single-phase liquid in the formation [1-4, 8]. This model includes:

- the differential equation of continuity of a single-phase filtration flow

$$\frac{\partial \varphi\rho}{\partial t} + div\vec{V}\rho = 0 \tag{1}$$

- the differential equation of motion, presented in the form of Darcy's filtration law

$$\vec{V} = -\frac{k}{\mu}\nabla p \tag{2}$$

- and the equations of state of oil and porous medium

$$\rho = \rho_o e^{c_f(p-p_o)}, \quad \varphi = \varphi_o e^{c_r(p-p_o)} \tag{3}$$

where p is the pressure, φ is the porosity coefficient, ρ is the density of the liquid, $\vec{V}$ is the velocity vector of oil in

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a porous medium, μ is the dynamic viscosity of oil, k is the absolute permeability of the porous medium, c_f is the compressibility coefficient of oil, c_r is the elasticity coefficient of the formation, ρ_0, φ_0 is the density of oil and the porosity coefficient at a fixed value of reservoir pressure $p=p_0$.

From the ratios (1)-(3), it is possible to obtain a three-dimensional mathematical model of the elastic regime of oil reservoir development in cylindrical coordinates (r, θ, z)

$$\beta\frac{\partial p}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k}{\mu}\frac{\partial p}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{k}{\mu}\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{k}{\mu}\frac{\partial p}{\partial z}\right) \qquad (4)$$

where $\beta=\varphi_0(c_f+c_r)$ is the coefficient of elastic capacity of the formation.

Obviously, in order to solve practical problems related to the development of reservoirs in an elastic regime, based on the model (4), it is necessary to know the initial and boundary conditions describing the initial state of the formation and the interaction of the formation with its environment. However, it should be noted that access to the oil reservoir is limited and is possible only through wells opening the reservoir in small areas, the initial pressure distribution in the reservoir, as well as the pressure or flow of oil at the outer boundary of the reservoir are not available for direct measurements. Therefore, it is practically impossible to accurately represent the initial condition in the formation and the condition at the outer boundary of the formation. The main sources of information about the processes occurring in the reservoir during development are production wells, where the pressure and flow rate of the well are available for direct measurements. In this regard, for the practice of developing oil reservoirs in an elastic mode, it is important to determine the pressure distribution in the reservoir only on the basis of information received from wells.

Problem statement and solution method

Let us consider a cylindrical oil-bearing reservoir with a thickness h and radius R, bounded from above and below by impermeable planes. In the center of the cylindrical formation there is an operational hydrodynamically perfect well of radius r_w, the axis of which coincides with the axis of the cylinder. The pressure at the bottom of the well exceeds the pressure of oil saturation with gas and the reservoir is developed in an elastic mode. Assuming, that the oil inflow from the reservoir to the well is a plane-radial flow of a single-phase liquid, the mathematical model of the elastic mode of development (4) for this single-well formation is represented as

$$\frac{\partial p(r,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(r)\frac{\partial p(r,t)}{\partial r}\right),\ r_w<r<R,\ 0<t\le T \qquad (5)$$

where

$$\lambda(r)=\frac{k(r)}{\mu\beta}$$

Suppose that the flow rate of the well and the pressure at the bottom of the well change over time according to given laws, i.e. for equation (5) we have the following conditions

$$2\pi r_w h\frac{k(r_w)}{\mu}\frac{\partial p(r_w,t)}{\partial r}=Q(t) \qquad (6)$$

$$p(r_w,t)=p_w(t) \qquad (7)$$

where $Q(t)$ and $p_w(t)$ are the specified functions.

However, due to the fact that the initial pressure distribution in the reservoir, as well as the pressure and fluid flow at the outer boundary of the reservoir, are not known, it is not possible to formulate the initial condition and boundary condition corresponding to the interaction of the reservoir with its surrounding area.

Thus, the problem of determining pressure fields in a reservoir is reduced to solving equation (5) when conditions (6), (7) are met. Problem (5)-(7) belongs to the class of boundary inverse problems without initial conditions [9, 10]. Numerous papers have been devoted to theoretical research and the development of numerical methods for solving inverse problems related to the identification of boundary conditions [9-18].

To numerically solve the inverse problem (5)-(7), first we discretize a given rectangular area $\{r_w\le r\le R,\ 0<t\le T\}$ in space and time, with steps $\Delta r=\frac{R-r_w}{n}$ in variable r and $\Delta t=\frac{T}{m}$ variable t, i.e. we construct a difference grid

$$\bar{\omega}=\left\{(r_i,t_j): r_i=r_w+i\Delta r,\ t_j=j\Delta t,\ i=0,1,2,...n,\ j=0,1,2,...m\right\}$$

Let's build a discrete model of the problem (5)-(7) on a grid $\bar{\omega}$, using the method of difference approximation. In this case, the derivative $\frac{\partial p(r,t)}{\partial t}$ in equation (5) is approximated by the difference «backwards» $\frac{\partial p(r,t)}{\partial t}\Big|_{r=r_i,t=t_j}\approx$ $\approx\frac{p(r_i,t_j)-p(r_i,t_{j-1})}{\Delta t}$. As a result, we will have

$$\frac{p_i^j-p_i^{j-1}}{\Delta t}=\frac{1}{r_i\Delta r}\left[r_{i+1/2}\lambda_{i+1/2}\frac{p_{i+1}^j-p_i^j}{\Delta r}-r_{i-1/2}\lambda_{i-1/2}\frac{p_i^j-p_{i-1}^j}{\Delta r}\right],$$

$$i=\overline{1,n-1},\quad j=\overline{1,m},$$

$$\frac{p_1^j-p_0^j}{\Delta r}=\frac{\mu}{2\pi r_w hk(r_w)}Q^j,\ \ j=\overline{0,m},$$

$$p_0^j=p_w^j,\quad j=\overline{0,m}$$

where $p_i^j\approx p(r_i,t_j)$, $r_{i\pm1/2}=r_i\pm\Delta r/2$, $\lambda_{i\pm1/2}=(\lambda(r_{i\pm1})+\lambda(r_i))/2$, $Q^j=Q(t_j)$

As can be seen, the discrete model of the problem (5)-(7) is a system of linear algebraic equations in which the unknowns are $p_i^j,\ i=\overline{1,n},\ j=\overline{0,m}$ i.e. approximate values of the desired function $p(r, t)$ at the nodal points of the grid $\bar{\omega}$.

The resulting system of linear algebraic equations is transformed to the form

$$a_i p_{i-1}^j-c_i p_i^j+b_i p_{i+1}^j=-\frac{p_i^{j-1}}{\Delta t},\ \ i=\overline{1,n-1},\ j=\overline{1,m}, \qquad (8)$$

$$p_1^j=p_0^j+\sigma Q^j,\ \ j=\overline{0,m}, \qquad (9)$$

$$p_0^j=p_w^j,\ \ j=\overline{0,m} \qquad (10)$$

where $a_i=\frac{r_{i-1/2}\lambda_{i-1/2}}{r_i\Delta r^2}$, $b_i=\frac{r_{i+1/2}\lambda_{i+1/2}}{r_i\Delta r^2}$, $c_i=a_i+b_i+\frac{1}{\Delta t}$,

$$\sigma=\frac{\mu\Delta r}{2\pi r_w hk(r_w)}$$

According to conditions (9), (10) on two spatial layers $r_0=r_w$ and $r_1=r_0+\Delta r$ (in the figure, the corresponding nodes are indicated by circles), the values of variables p_0^j and p_1^j are considered set. Now the task consists in finding approximate values of the desired function $p(r, t)$ from the system of equations (8)-(10) at the remaining nodal points, i.e. $p_i^j,\ i=\overline{2,n},\ j=\overline{0,m}$.

A simple analysis of the system of equations (8)–(10) shows that from this system it is consistently possible to determine:

$$p_2^j, j=\overline{1,l}, \quad \text{at } i=1$$
$$p_3^j, j=\overline{2,l}, \quad \text{at } i=2$$
$$p_4^j, j=\overline{3,l}, \quad \text{at } i=3$$
$$p_5^j, j=\overline{4,l}, \quad \text{at } i=4$$
....
$$p_n^j, j=l, i=n-1$$

where $l=n-1$. In the figure, the grid nodes corresponding to the calculated variables are indicated by crosses. At the same time, as shown in the figure, in some nodes located in the triangular region D, it is not possible to find the value of the desired function.

The numerical implementation of the above calculation procedure when setting p_0^j and p_0^j can be carried out using the recurrent formula

$$p_{i+1}^j = \frac{c_i p_i^j - a_i p_{i-1}^j}{b_i} - -\frac{p_i^{j-1}}{b_i \Delta t}, \quad j=\overline{1,l}, \quad i=\overline{1,j} \tag{11}$$

Thus, formula (11) allows us to calculate approximate values of the desired function $p(r, t)$ at the nodal points of the grid, including nodes on the time layer $j=l$, with the exception of unmarked nodes in the D region.

Now let's move on to determining the approximate values of the desired function $p(r, t)$ at the nodal points lying above the time layer $j=l$ from the system of equations (8)-(10). The systems of equations (8)-(10) are written as

$$a_i p_{i-1}^j - c_i p_i^j + b_i p_{i+1}^j = -\frac{p_i^{j-1}}{\Delta t}, \tag{12}$$
$$i=\overline{1,n-1}, \quad j=\overline{l+1, m}$$

$$p_1^j = p_0^j + \sigma Q^j, \quad j=\overline{l,m} \tag{13}$$

$$p_0^j = p_w^j, \quad j=\overline{l,m} \tag{14}$$

where p_i^l, $i=\overline{0,n}$ are considered famous.

To solve a system of linear algebraic equations (12)-(14), we use the idea of decomposing this system into mutually independent subsystems, each of which can be solved independently of each other [10, 16]. For this purpose, the solution of the system of equations (12)-(14) for each fixed value $j=l+1, l+2, \ldots m$ is represented as

$$p_i^j = u_i^j + p_n^j w_i^j, \quad i=\overline{0,n} \tag{15}$$

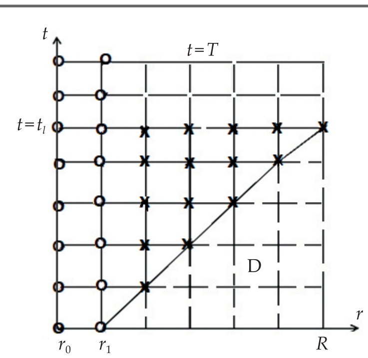

Fig. The discrete domain of the numerical solution of the inverse problem, $t_l=l\Delta t$

where u_i^j, w_i^j and p_n^j are unknown variables. Substituting the ratio (15) into (12) and (13), we will have

$$\left[a_i u_{i-1}^j - c_i u_i^j + b_i u_{i+1}^j + \frac{p_i^{j-1}}{\Delta t}\right] + p_n^j\left[a_i w_{i-1}^j - c_i w_i^j + b_i w_{i+1}^j\right] = 0,$$
$$\left[u_1^j - u_0^j - \sigma Q^j\right] + p_n^j\left[w_1^j - w_0^j\right] = 0$$

Obviously, these ratios will be performed automatically for each fixed value j, $j=l+1, l+2, \ldots m$, if:

- the variables satisfy a system of linear algebraic equations

$$a_i u_{i-1}^j - c_i u_i^j + b_i u_{i+1}^j = -\frac{p_i^{j-1}}{\Delta t}, \quad i=\overline{1,n-1} \tag{16}$$

$$u_1^j = u_0^j + \sigma Q^j \tag{17}$$

$$u_n^j = 0 \tag{18}$$

- and the variables w_i^j satisfy the following system of linear algebraic equations

$$a_i w_{i-1}^j - c_i w_i^j + b_i w_{i+1}^j = 0, \quad i=\overline{1,n-1} \tag{19}$$

$$w_1^j = w_0^j \tag{20}$$

$$w_n^j = 1 \tag{21}$$

The obtained independent systems of equations (16)-(18) and (19)-(21) for each fixed value $j=l+1, l+2, \ldots m$ represent a system of linear algebraic equations with a tridiagonal matrix, the solutions of which are determined by the Thomas method [10]. Substituting representation (15) into (14), we will have

$$u_0^j + p_n^j w_0^j = p_w^j$$

From here we obtain a formula for determining the approximate value p_n^j of the desired function $p(r, t)$ at the outer boundary of the formation $r=R$

$$p_n^j = \frac{p_w^j - u_0^j}{w_0^j} \tag{22}$$

Having determined the values p_n^j, it is possible to calculate the approximate values of the desired function $p(r, t)$ on the time layer j using formula (15) at $r=r_i$, i.e. p_i^j, $i=\overline{1,n-1}$.

Thus, the proposed numerical method for solving the boundary inverse problem without the initial condition (5)–(7) makes it possible to calculate pressure distributions in a single-well reservoir developed in an elastic mode at each time layer.

A numerical example

The proposed computational algorithm has been tested for data from a model single-well oil reservoir with the following characteristics:

- formation radius R=100 m;
- the thickness of the formation h=0 m;
- the radius of the well r_w=0.1 m;
- the coefficient of absolute permeability of the formation $k=1.5\cdot10^{-12}$ m^2;
- initial reservoir pressure 100 atm.;
- well flow rate $Q(t)$=60 m^3/day;
- dynamic viscosity of oil $\mu=3\cdot10^{-3}$ Pa$\cdot$s;
- the coefficient of elastic capacity of the formation $\beta=2\cdot10^{-10}$ Pa^{-1};

The numerical experiment was carried out according to the following scheme:

- I. A solution of equation (5) is determined that satisfies the boundary condition at the well (6), the condition of non-flow at the outer boundary of the formation $\frac{\partial p(R,t)}{\partial r}=0$ and the specified initial condition (the direct problem is solved);
- II. The found dependence $p_w(t)=p(r, t)$, $r=r_w$ is taken as the exact input data for the numerical solution of the inverse problem (5)-(7) for restoring the pressure distribution in the reservoir.

The calculations were carried out on a uniform space-time difference grid with steps of Δx=4.995 m, Δt=60 s. The results of numerical experiments to determine the dynamics of pressure distribution in the reservoir are presented in the table. It contains t – the time, r – the distance from the center of the well.

Calculations have shown that the dynamics of the pressure distribution in the reservoir, with the exception of the area D indicated in the figure, is restored absolutely accurately. Therefore, the exact dynamics of the pressure distribution in the reservoir, determined during the solution of the direct problem, is not presented in the table.

Analysis of the results of numerical experiments indicates that with the beginning of reservoir development, the pressure distribution is restored in a time-varying region. For example, 10 minutes after the start of reservoir development, the pressure distribution is restored within a radius of 50.05 meters around the well. And 20 minutes after the start of reservoir development, the pressure distribution is completely restored in the reservoir.

Table

Computed pressure distributions in the reservoir

r, m	**Pressure, atm.**					
	***t*=2 min.**	***t*=2 min.**	***t*=2 min.**	***t*=2 min.**	***t*=2 min.**	***t*=2 min.**
0.100	80.29	77.37	75.26	73.97	72.93	71.98
5.095	91.33	88.41	86.30	85.01	83.97	83.02
10.090	94.85	92.11	90.04	88.77	87.73	86.78
15.085		94.26	92.26	91.00	89.97	89.02
20.080		95.70	93.80	92.56	91.54	90.60
25.075		96.74	94.96	93.75	92.74	91.80
30.070			95.86	94.69	93.69	92.75
35.065			96.58	95.45	94.47	93.53
40.060			97.16	96.08	95.11	94.18
45.055			97.64	96.60	95.65	94.72
50.050			98.04	97.04	96.10	95.18
55.045				97.40	96.48	95.57
60.040				97.71	96.80	95.89
65.035				97.95	97.06	96.16
70.030				98.16	97.27	96.37
75.025				98.32	97.44	96.55
80.020					97.58	96.68
85.015					97.67	96.78
90.010					97.73	96.84
95.005					97.76	96.87
100.00					97.76	96.87

Conclusions

The problem of determining the dynamics of pressure distribution during the development of a single-well formation in an elastic mode, according to the specified pressure at the bottom and the well's flow rate is considered. The model of this problem is presented in the form of a boundary inverse problem without initial conditions for the equation of a nonstationary plane-radial filtration flow of a single-phase liquid in a reservoir. Computational algorithm, proposed for the numerical solution of the problem:

1. is based on the discretization of the problem by the method of difference approximation;
2. allows for a certain period of time to restore the current reservoir pressure;
3. allows you to control the dynamics of pressure distribution in the reservoir, only on the basis of available information about the well flow rate and bottom-hole pressure.

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