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CALCULATION OF METAL ELEMENTS DEFLECTION USING A THREE-LINE STRAIN DIAGRAM

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A B S T R A C T

In the article, using a three-line symmetrical strain diagrams for metal, a technique is given for constructing a «moment-curvature» diagram, which is important for determining the deflections of steel beams. Approximations of the moment-curvature diagram by polynomials of the third and fifth degrees, which have high accuracy, are proposed. In numerical examples, it was established that when plastic deformations develop, it is not permissible to determine deflections using a linear model. In addition, numerical examples show that physical nonlinearity has little effect on internal forces in statically indeterminate systems, but has a strong effect on the stress-strain state.

Keywords: beam; physical nonlinearity; moment-curvature diagram; deflection.

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The advanced theory for structural elements design calculation is based on real deformation diagrams of materials [3-5]. This is due to the fact that only on the basis of calculation methods based on nonlinear diagrams is it possible to determine the hidden strength and deformation reserves of structures and their elements. Standard methods for calculating steel structures [1,2] do not allow the most accurate determination of deflections taking into account the development of plastic deformations. In order to determine them, in most cases, various numerical methods are used [6-9]. It should be noted that, along with numerical methods, the presence of simple calculation formulas that complement the standard calculation methodology opens up great opportunities for the argumentation of the final options for predicting deformations of steel structures. Using a three-linear deformation diagram for the steel, a method for constructing a moment-curvature diagram was created and its use for determining the deflections of beams was shown.

The deformation diagram is curvilinear for high-tensile-strength steel. Such diagrams can be approximated with high accuracy by three-line diagrams. Therefore, calculations based on such diagrams offer the prospect of obtaining the results consistent with the results of experimental studies. A schematic three-line diagram of steel deformation is shown in figure 1.

Analytically this diagram is described as follows:

$$\sigma_s = \begin{cases} E_s \cdot \varepsilon_s\,; \quad \text{at } \left|\varepsilon_s\right| \le \varepsilon_{s1} \\ \sigma_{s1} + \dfrac{\sigma_{s2}-\sigma_{s1}}{\varepsilon_{s2}-\varepsilon_{s1}} \cdot \left(\varepsilon_s - \varepsilon_{s1}\right); \ \text{at } \varepsilon_{s1} < \left|\varepsilon_s\right| \le \varepsilon_{s2} \\ \sigma_{s3}\,; \ at \ \varepsilon_{s2} < \left|\varepsilon_s\right| \le \varepsilon_{s3} \end{cases} \tag{1}$$

Figure 1 shows the construction of a moment-curvature diagram using for an element with a rectangular cross-section. Based on the hypothesis of plane sections, for the distribution of deformations along the height of the section we

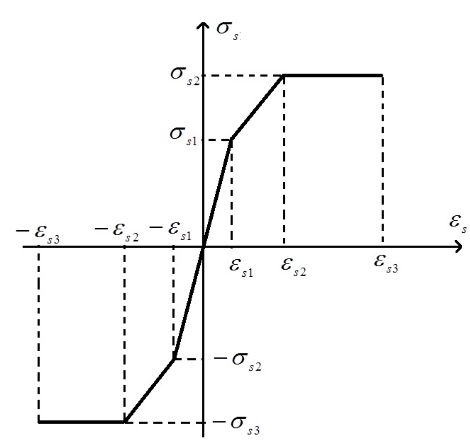

Fig. 1. Three-line diagram of steel deformation

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have $\varepsilon_{sz} = \dfrac{2\varepsilon_s}{h} \cdot z = \chi \cdot z$, here χ- is the curvature of the section. Depending on the level of the current load, options for the distribution of normal stresses over the section are possible, shown in figure 2. When the section operates within the limits of elasticity, based on the formulas for the resistance of materials, we have:

$$M_s = \int_A \sigma_z \cdot z \cdot dAE_s = b \cdot \int_{-h/2}^{h/2} E_z \cdot \frac{2\varepsilon_s}{h} \cdot z \cdot zdz = \\ = \frac{b \cdot h^2}{6} \cdot E_s \cdot \varepsilon_s = \frac{b \cdot h^3}{12} \cdot E_s \cdot \chi = E_s \cdot J \cdot \chi \tag{2}$$

Similarly, for the second case of voltage distribution we can write:

$$M_s = \int_A \sigma_z \cdot z \cdot dAE_s = b \cdot \int_{-z_1}^{z_1} E_z \cdot \frac{2\varepsilon_s}{h} \cdot z \cdot zdz + \\ + 2b \cdot \int_{z_1}^{h/2} \left[\sigma_{s1} + \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \left(\frac{2\varepsilon_s}{h} \cdot z - \varepsilon_{s1} \right) \right] \cdot zdz = \\ = \frac{4b \cdot z_1^3}{3h} \cdot E_s \cdot \varepsilon_s + b \cdot \left(\sigma_{s1} - \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_{s1} \right) \cdot \left(\frac{h^2}{4} - z_1^2 \right) + \\ + \frac{4b}{3h} \cdot \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_s \cdot \left(\frac{h^3}{8} - z_1^3 \right) \tag{3}$$

Here $z_1 = \dfrac{\varepsilon_{s1} \cdot h}{2\varepsilon_s} = \dfrac{\varepsilon_{s1}}{\chi}$. Based on this, finally, for the case under consideration, the expression of the moment can be represented as:

$$M_s = \frac{2b}{3} \cdot E_s \cdot \frac{\varepsilon_{s1}^3}{\chi^2} + b \cdot \left(\sigma_{s1} - \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_{s1} \right) \cdot \left(\frac{h^2}{4} - \frac{\varepsilon_{s1}^2}{\chi^2} \right) + \\ + \frac{2b}{3} \cdot \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \chi \cdot \left(\frac{h^3}{8} - \frac{\varepsilon_{s1}^3}{\chi^3} \right) \tag{4}$$

This dependence can be more briefly represented as:

$$M_s = A_1 + B_1 \cdot \chi + \frac{C_1}{\chi^2} \tag{5}$$

Where the following notations are introduced for constant quantities:

$$A_1 = \frac{b \cdot h^2}{4} \cdot \left(\sigma_{s1} - \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_{s1} \right); \; B_1 = \frac{b \cdot h^3}{12} \cdot \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}}; \\ C_1 = \frac{2b}{3} \cdot E_s \cdot \varepsilon_{s1}^3 - b \cdot \left(\sigma_{s1} - \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_{s1} \right) \cdot \varepsilon_{s1}^2 - \\ - \frac{2b}{3} \cdot \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_{s1}^3 \tag{6}$$

Proceeding similarly for the third case of distribution of normal stresses, the following dependence for the bending moment was found:

$$M_s = A_2 + \frac{C_2}{\chi^2} \tag{7}$$

Where the following notations are introduced for constant quantities:

$$A_2 = \frac{b \cdot h^2}{4} \cdot \sigma_{s2}; \\ C_2 = \frac{2b}{3} \cdot E_s \cdot \varepsilon_{s1}^3 + b \cdot \left(\sigma_{s1} - \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \varepsilon_{s1} \right) \cdot \left(\varepsilon_{s2}^2 - \varepsilon_{s1}^2 \right) + \\ + \frac{2b}{3} \cdot \frac{\sigma_{s2} - \sigma_{s1}}{\varepsilon_{s2} - \varepsilon_{s1}} \cdot \left(\varepsilon_{s2}^3 - \varepsilon_{s1}^3 \right) - b \cdot \sigma_{s2} \cdot \varepsilon_{s2}^2 \tag{8}$$

Combining all the above expressions, we can write the following dependence of the bending moment on curvature in the case of describing the metal deformation diagram with a three-line diagram:

$$M_s = \begin{cases} E_s \cdot J \cdot \chi ; \quad \text{at} \quad |\chi| \le \chi_1 = 2\varepsilon_{s1} / h \\ A_1 + B_1 \cdot \chi + \dfrac{C_1}{\chi^2}; \; \text{at} \;\; \chi_1 = 2\varepsilon_{s1} / h < |\chi| \le \chi_2 = 2\varepsilon_{s2} / h \\ A_2 + \dfrac{C_2}{\chi^2}; \quad \text{at} \quad \chi_2 = 2\varepsilon_{s2} / h < |\chi| \le \chi_3 = 2\varepsilon_{s3} / h \end{cases} \tag{9}$$

As an example, in figure 3, with ε_{s1}=0.0012; ε_{s2}=0.0025; ε_{s3}=0.0035; σ_{s1}=240 MPa; ε_{s2}=350 MPa; E_s=200000 MPa moment-curvature diagrams are given for various section sizes. For solving practical problems, especially when calculating deflections of metal beams, the dependence of curvature on the bending moment is convenient. Therefore, we take the analytical approximation in the form of the following cubic parabola:

$$\chi = \alpha \cdot M + \beta \cdot M^3 \tag{10}$$

The unknown parameters α and β are found from the condition that when $\chi=\chi_3=\chi_u$ the condition $M=M_3=M_u$ is met and the sum of the areas of curvilinear triangles under curves (9) and (10) should be equal to $\chi_u \cdot M_u$. For the mentioned areas we have:

$$\Omega_\chi = \int_0^{M_u} \left(\alpha \cdot M + \beta \cdot M^3 \right) dM = \frac{\alpha \cdot M_u^2}{2} + \frac{\beta \cdot M_u^4}{4}; \\ \Omega_M = \frac{b \cdot h^3}{12} \cdot E_s \cdot \int_0^{\chi_1} \chi d\chi + \int_{\chi_1}^{\chi_2} \left(A_1 + B_1 \cdot \chi + \frac{C_1}{\chi^2} \right) d\chi + \\ + \int_{\chi_2}^{\chi_3} \left(A_2 + \frac{C_2}{\chi^2} \right) d\chi = \frac{b \cdot h^3}{24} \cdot E_s \cdot \chi_1^2 + A_1 \cdot \left(\chi_2 - \chi_1 \right) + \\ + \frac{B_1 \cdot \left(\chi_2^2 - \chi_1^2 \right)}{2} - \frac{C_1}{\chi_2} + \frac{C_1}{\chi_1} + A_2 \cdot \left(\chi_3 - \chi_2 \right) - \frac{C_2}{\chi_3} + \frac{C_2}{\chi_2} \tag{11}$$

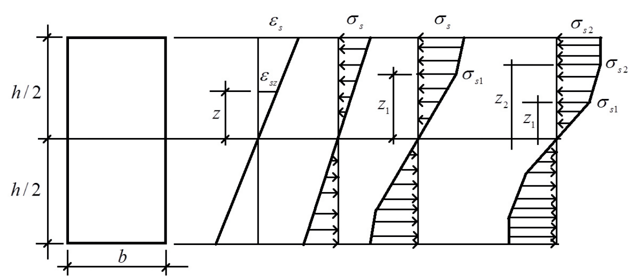
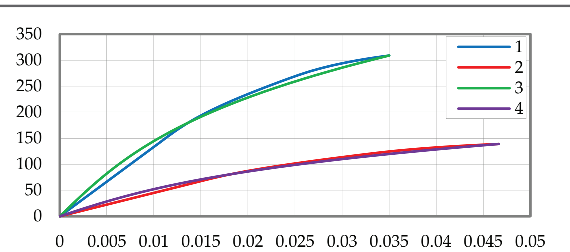


Fig. 2. Design diagram of the section



1–$b \times h$=0.1×0.2 m; 3– $b \times h$=0.1×0.2 m cubic approximation;
2–$b \times h$=0.08×0.15 m; 4–$b \times h$=0.08×0.15 m cubic approximation

Fig. 3. Moment-curvature diagrams

In this case, the system of equations for determining the unknowns becomes:

$$\begin{cases} \alpha \cdot M_u + \beta \cdot M_u^3 = \chi_3 \\ \dfrac{\alpha \cdot M_u^2}{2} + \dfrac{\beta \cdot M_u^4}{4} = \chi_3 \cdot M_u - \Omega_M \end{cases}$$

Having solved this system we get,

$$\alpha = \frac{3 \cdot \chi_3}{M_u} - \frac{4 \cdot \Omega_M}{M_u^2}; \quad \beta = -\frac{2 \cdot \chi_3}{M_u^3} + \frac{4 \cdot \Omega_M}{M_u^4} \qquad (12)$$

By way of comparison, figure 3 also shows graphs obtained from approximation dependencies (10). As one can see, the cubic approximation quite accurately describes the moment-curvature diagram. We will show the application of the obtained expressions to determine the deflections of statically determinate beams. Let's consider the beam shown in figure 4. Based on (10), we create a differential equation for deflections. In dimensionless coordinates, this equation has the form:

$$y''(\eta) = -y_0 \cdot \left(\eta^3 - 3 \cdot \eta^2 + 2 \cdot \eta + \delta \cdot \left(\eta^3 - 3 \cdot \eta^2 + 2 \cdot \eta\right)^3\right)$$

The following notations are introduced here: $y_0 = \alpha \cdot \dfrac{ql^4}{6}$ and $\delta = \beta \cdot \dfrac{q^2 l^4}{36 \cdot \alpha}$.

After appropriate simplifications we have:

$$y''(\eta) = -y_0 \cdot \left(\eta^3 - 3 \cdot \eta^2 + 2 \cdot \eta + \right.$$
$$\left. + \delta \cdot \left(\eta^9 - 9\eta^8 + 33\eta^7 - 63\eta^6 + 66\eta^5 - 36\eta^4 + 8\eta^3\right)\right)$$

Integrating both sides of this dependence twice, we find:

$$y'(\eta) = C_1 - y_0 \cdot \left(\frac{\eta^4}{4} - \eta^3 + \eta^2 + \delta \times \right.$$
$$\left. \times \left(\frac{\eta^{10}}{10} - \eta^9 + \frac{33\eta^8}{8} - 9\eta^7 + 11\eta^6 - \frac{36\eta^5}{5} + 2\eta^4\right)\right)$$

$$y(\eta) = C_1 \cdot \eta + C_2 - y_0 \cdot \left(\frac{\eta^5}{20} - \frac{\eta^4}{4} + \frac{\eta^3}{3} + \delta \times \right.$$
$$\left. \times \left(\frac{\eta^{11}}{110} - \frac{\eta^{10}}{10} + \frac{11\eta^9}{24} - \frac{9\eta^8}{8} + \frac{11\eta^7}{7} - \frac{6\eta^6}{5} + \frac{2\eta^5}{5}\right)\right)$$

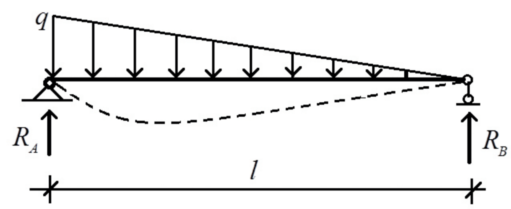


Fig. 4. Design diagram of the beam

According to the boundary conditions when η=0 and η=1 and the condition y=0 is satisfied. From these conditions it follows that, C_2=0 and $C_1 = y_0 \cdot \left(\dfrac{2}{15} + \dfrac{16\delta}{1155}\right)$. Then, finally, for the deflection function we have:

$$y(\eta) = y_0 \cdot \left[\left(\frac{2}{15} + \frac{16\delta}{1155}\right) \cdot \eta - \frac{\eta^5}{20} + \frac{\eta^4}{4} - \frac{\eta^3}{3} - \right.$$
$$\left. - \delta \cdot \left(\frac{\eta^{11}}{110} - \frac{\eta^{10}}{10} + \frac{11\eta^9}{24} - \frac{9\eta^8}{8} + \frac{11\eta^7}{7} - \frac{6\eta^6}{5} + \frac{2\eta^5}{5}\right)\right] \qquad (13)$$

The maximum ordinate of the moment diagram is obtained in the section at a distance $x_0 = \eta_0 \cdot l = \dfrac{3 - \sqrt{3}}{3} \cdot l$ from the left support and its value is determined as $M_{\max} = \dfrac{ql^2}{6} \cdot (1 - \eta_0) \cdot (2\eta_0 - \eta_0^2) = \dfrac{ql^2}{6\sqrt{3}}$. This means that the corresponding value of the distributed load is defined as $q_u = \dfrac{6\sqrt{3} \cdot M_u}{l^2}$. Then the maximum value of the nonlinearity parameter will be

$$\delta_u = \beta \cdot \frac{q^2 l^4}{36 \cdot \alpha} = \frac{\beta}{36 \cdot \alpha} \cdot \frac{36 \cdot 3 \cdot M_u^2}{l^4} \cdot l^4 = \frac{3 \cdot \beta}{\alpha} \cdot M_u^2$$

Let's consider a numerical example, assuming that b=0.1 m; h=0.2 m; E_s=200000 MPa; σ_{s1}=240 MPa; σ_{s2}=350 MPa; ε_{s1}=0.0012; ε_{s2}=0.0025; ε_{s3}=0.0035. With these data we have M_u=308.59864 kN·m; α=0.5677·10^{-4} (kN·m^2)$^{-1}$; β=0.594782·10^{-9} (kN^3·m^4)$^{-1}$. For different values of the nonlinearity parameter, the ordinates of the deflection

Table 1

The ordinate values of the deflection diagram for different values of the nonlinearity parameter

η	$\dfrac{y}{y_0}$; $\delta_u = 2.99328585$					
	δ=0	δ=0.2δ_u	δ=0.4δ_u	δ=0.6δ_u	δ=0.8δ_u	δ=δ_u
0	0	0	0	0	0	0
0.1	0.01302	0.01385	0.01468	0.01551	0.01633	0.01716
0.2	0.02438	0.02600	0.02762	0.02924	0.03085	0.03247
0.3	0.03290	0.03517	0.03743	0.03969	0.04196	0.04422
0.4	0.03789	0.04053	0.04317	0.04581	0.04846	0.05110
0.5	0.03906	0.04175	0.04444	0.04713	0.04982	0.05251
0.6	0.03651	0.03894	0.04137	0.04379	0.04622	0.04864
0.7	0.03062	0.03256	0.03450	0.03644	0.03837	0.04031
0.8	0.02202	0.02334	0.02467	0.02599	0.02732	0.02864
0.9	0.01150	0.01217	0.01283	0.01350	0.01417	0.01484
1.0	0	0	0	0	0	0

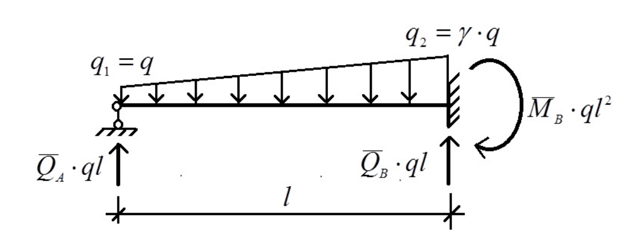


Fig. 5. Design diagram of a statically indeterminate beam

diagram were calculated, which are shown in table 1. As can be seen from table 1, nonlinearity greatly affects the magnitude of the deflections. For example, for the beam under consideration in the limit state, the maximum deflection is 34.5% greater than the corresponding elastic deflection of the beam under consideration.

Now we will show the application of the obtained dependencies to the calculation of statically indeterminate beams (fig. 5). Taking the origin of coordinates at the hinge support to change the bending moment along the length of the beam, we can write:

$$M = \overline{Q}_A \cdot ql \cdot x - \frac{qx^2}{2} - \frac{1}{6}\cdot(\gamma - 1)\cdot\frac{qx^3}{l}$$

Then the differential equation for beam bending will be written in the following form

$$y'' = -\alpha\cdot\left(\overline{Q}_A \cdot ql \cdot x - \frac{qx^2}{2} - \frac{1}{6}\cdot(\gamma-1)\cdot\frac{qx^3}{l}\right) - \\ -\beta\cdot\left(\overline{Q}_A \cdot ql \cdot x - \frac{qx^2}{2} - \frac{1}{6}\cdot(\gamma-1)\cdot\frac{qx^3}{l}\right)^3$$

Passing to dimensionless coordinates in this equation we obtain:

$$y''_\eta = -\alpha\cdot ql^4\cdot\left[\overline{Q}_A\cdot\eta - \frac{\eta^2}{2} - \frac{1}{6}\cdot(\gamma-1)\cdot\eta^3 + \right. \\ \left. +\delta\cdot\left(\overline{Q}_A\cdot\eta - \frac{\eta^2}{2} - \frac{1}{6}\cdot(\gamma-1)\cdot\eta^3\right)^3\right]$$

After appropriate simplifications we have:

$$\frac{y''_\eta}{\alpha\cdot ql^4} = -\overline{Q}_A\cdot\eta + \frac{\eta^2}{2} + \frac{1}{6}\cdot(\gamma-1)\cdot\eta^3 + \delta\cdot\frac{(\gamma-1)^3}{216}\cdot\eta^9 + \\ +\delta\cdot\frac{(\gamma-1)^2}{24}\cdot\eta^8 - \delta\cdot\left(\frac{(\gamma-1)^2}{12}\cdot\overline{Q}_A - \frac{\gamma-1}{8}\right)\cdot\eta^7 - \\ -\delta\cdot\left(\frac{\gamma-1}{2}\cdot\overline{Q}_A - \frac{1}{8}\right)\cdot\eta^6 - \delta\cdot\left(\frac{3}{4}\cdot\overline{Q}_A - \frac{\gamma-1}{2}\cdot\overline{Q}_A^2\right)\cdot\eta^5 + \\ +\frac{3}{2}\cdot\delta\cdot\overline{Q}_A^2\cdot\eta^4 - \overline{Q}_A^3\cdot\delta\cdot\eta^3$$

Integrating both sides of the last equation twice we get:

$$\frac{y'_\eta}{\alpha\cdot ql^4} = C_1 - \frac{\overline{Q}_A}{2}\cdot\eta^2 + \frac{\eta^3}{6} + \frac{\gamma-1}{24}\cdot\eta^4 + \delta\cdot\frac{(\gamma-1)^3}{2160}\cdot\eta^{10} + \\ +\delta\cdot\frac{(\gamma-1)^2}{216}\cdot\eta^9 - \delta\cdot\left(\frac{(\gamma-1)^2}{96}\cdot\overline{Q}_A - \frac{\gamma-1}{64}\right)\cdot\eta^8 - \\ -\delta\cdot\left(\frac{\gamma-1}{14}\cdot\overline{Q}_A - \frac{1}{56}\right)\cdot\eta^7 - \delta\cdot\left(\frac{1}{8}\cdot\overline{Q}_A - \frac{\gamma-1}{12}\cdot\overline{Q}_A^2\right)\cdot\eta^6 + \\ +\frac{3}{10}\cdot\delta\cdot\overline{Q}_A^2\cdot\eta^5 - \frac{\overline{Q}_A^3\cdot\delta}{4}\cdot\eta^4$$

$$\frac{y_\eta}{\alpha\cdot ql^4} = C_1\cdot\eta + C_2 - \frac{\overline{Q}_A}{6}\cdot\eta^3 + \frac{\eta^4}{24} + \frac{\gamma-1}{120}\cdot\eta^5 + \delta\cdot\frac{(\gamma-1)^3}{23760}\cdot\eta^{11} + \\ +\delta\cdot\frac{(\gamma-1)^2}{2160}\cdot\eta^{10} - \delta\cdot\left(\frac{(\gamma-1)^2}{864}\cdot\overline{Q}_A - \frac{\gamma-1}{576}\right)\cdot\eta^9 - \\ -\delta\cdot\left(\frac{\gamma-1}{112}\cdot\overline{Q}_A - \frac{1}{448}\right)\cdot\eta^8 - \delta\cdot\left(\frac{1}{56}\cdot\overline{Q}_A - \frac{\gamma-1}{84}\cdot\overline{Q}_A^2\right)\cdot\eta^7 + \\ +\frac{1}{20}\cdot\delta\cdot\overline{Q}_A^2\cdot\eta^6 - \frac{\overline{Q}_A^3\cdot\delta}{20}\cdot\eta^5$$

According to the boundary conditions, when η=0 then it must be y_η=0, and when η=1 then the condition y_η=0 and $y'_\eta = 0$ must be satisfied. From these conditions it follows that C_2=0;

$$C_1 = \alpha\cdot ql^4\cdot\overline{C}_1 = \alpha\cdot ql^4\cdot\left(\frac{\delta}{20}\cdot\overline{Q}_A^3 - \overline{Q}_A^2\cdot\lambda_1 + \overline{Q}_A\cdot\lambda_2 - \lambda_3\right) \quad (14)$$

$$C_1 = \alpha\cdot ql^4\cdot\overline{C}_1 = \alpha\cdot ql^4\cdot\left(\frac{\delta}{4}\cdot\overline{Q}_A^3 - \overline{Q}_A^2\cdot\omega_1 + \overline{Q}_A\cdot\omega_2 - \omega_3\right) \quad (15)$$

The following notations are introduced here:

$$\lambda_1 = \delta\times\left(\frac{\gamma-1}{84} + \frac{1}{20}\right);\ \lambda_2 = \delta\times\left(\frac{1}{56} + \frac{\gamma-1}{112} + \frac{(\gamma-1)^2}{864}\right) + \frac{1}{6}; \\ \lambda_3 = \delta\times\left(\frac{(\gamma-1)^3}{23760} + \frac{(\gamma-1)^2}{2160} + \frac{\gamma-1}{576} + \frac{1}{448}\right) + \frac{1}{24} + \frac{\gamma-1}{120}; \\ \omega_1 = \delta\cdot\left(\frac{\gamma-1}{12} + \frac{3}{10}\right);\ \omega_2 = \delta\cdot\left(\frac{1}{8} + \frac{\gamma-1}{14} + \frac{(\gamma-1)^2}{96}\right) + \frac{1}{2}; \\ \omega_3 = \delta\cdot\left(\frac{(\gamma-1)^3}{2160} + \frac{(\gamma-1)^2}{216} + \frac{\gamma-1}{64} + \frac{1}{56}\right) + \frac{1}{6} + \frac{\gamma-1}{24} \quad (16)$$

Then finally for the deflection function we obtain

$$\frac{y_\eta}{\alpha\cdot ql^4} = b_1\cdot\eta - b_3\cdot\eta^3 + b_4\cdot\eta^4 + b_5\cdot\eta^5 + b_6\cdot\eta^6 - \\ -b_7\cdot\eta^7 + b_8\cdot\eta^8 - b_9\cdot\eta^9 + b_{10}\cdot\eta^{10} + b_{11}\cdot\eta^{11} \quad (17)$$

Here, to simplify the notation, the following notations have been introduced:

$$b_1 = \overline{C}_1;\ b_3 = \frac{\overline{Q}_A}{6};\ b_4 = \frac{1}{24};\ b_5 = \frac{\gamma-1}{120} - \delta\cdot\frac{\overline{Q}_A^3}{20}; \\ b_6 = \frac{\delta}{20}\cdot\overline{Q}_A^2;\quad b_7 = \delta\cdot\left(\frac{1}{56}\cdot\overline{Q}_A - \frac{\gamma-1}{84}\cdot\overline{Q}_A^2\right); \\ b_8 = \delta\cdot\left(\frac{1}{448} - \frac{\gamma-1}{112}\cdot\overline{Q}_A\right);\ b_9 = \delta\cdot\left(\frac{(\gamma-1)^2}{864}\cdot\overline{Q}_A - \frac{\gamma-1}{576}\right); \\ b_{10} = \delta\cdot\frac{(\gamma-1)^2}{2160};\ b_{11} = \delta\cdot\frac{(\gamma-1)^3}{23760} \quad (19)$$

From equations (14) and (15) the following cubic equation follows for determining the unknown support reaction:

$$\delta\cdot\overline{Q}_A^3 - 5\cdot(\omega_1 - \lambda_1)\cdot\overline{Q}_A^2 + \\ +5\cdot(\omega_2 - \lambda_2)\cdot\overline{Q}_A - 5\cdot(\omega_3 - \lambda_3) = 0 \quad (20)$$

From (20) it is clear that in physically nonlinear statically indeterminate systems, the disclosure of static indetermination leads to complex calculations. Therefore, in such systems, the creation of analytical calculation methods is practically unrealistic. Even in the considered simple case of a once statically indeterminate system, the disclosure of static indetermination is reduced to solving a cubic equation. It is clear that with an increase in the degree of static indetermination, the calculation algorithm becomes much more complicated. At the same time, the use of nonlinear calculation models allows not only to most reliably determine the stress-strain state and to make the most maximum and reliable use of the strength reserves of materials. As can be seen from the above dependencies, the solution of physically nonlinear problems is possible only with the use of computer programs. For the beam under consideration in the limit state, plastic hinges are formed on the right embedment and at a certain distance from the left support. Given a coordinate, it is determined as the solution to the following cubic equation:

$$2(\gamma-1)\cdot\eta_0^3+3\gamma\cdot\eta_0^2+6\eta_0-\gamma-2=0 \tag{21}$$

In this case, the limiting moment is calculated using the formula:

$$\frac{M_{ult}}{ql^2}=\frac{(\gamma+2)\cdot\eta_0-3\cdot\eta_0^2-(\gamma-1)\cdot\eta_0^3}{6\cdot(1+\eta_0)} \tag{22}$$

When γ=2 from (21) it follows that η_0=0.44225 according to (22) the limiting moment is determined by the dependence $M_{ult}=0.12662\cdot q_u\cdot l^2$. Then for the maximum value of the nonlinearity parameter we obtain:

$$\delta_u=\beta\cdot\frac{q^2l^4}{36\cdot\alpha}=\frac{\beta}{36\cdot\alpha}\cdot\frac{M_u^2}{0.12662^2}=$$
$$=\frac{0.5948\cdot10^{-9}}{36\cdot0.5677\cdot10^{-4}}\cdot\frac{308.5986^2}{0.12662^2}=1.740$$

An appropriate software module has been compiled that implements the calculation of the ordinates of the deflection diagram, and using this program, calculations have been carried out at various load levels, the results of which are given in table 2.

For the example under consideration, physical nonlinearity does not actually affect static indetermination. In addition, the influence of physical nonlinearity in statically indeterminate systems is less than in statically determinate systems [10, 11]. Unlike linear systems, it is impossible to give general recommendations in nonlinear systems. Conclusions must be drawn on a case-by-case basis based on appropriate calculations.

Sometimes, to improve accuracy, it is necessary to approximate the moment-curvature diagrams with a fifth-degree polynomial as follows:

$$\chi=\alpha\cdot M+\beta\cdot M^3+\gamma\cdot M^5 \tag{23}$$

In order to determine the unknown parameters included in this dependence, in addition to those used above, when determining the parameters of a cubic parabola, the condition is additionally used that the product of the tangents of the angles formed by the tangents drawn at the origin of the given curve and the approximating curve is equal to one. Analytically, these conditions are written as follows

$$\begin{cases}\alpha\cdot M_u+\beta\cdot M_u^3+\gamma\cdot M_u^5=\chi_3\\ \alpha\cdot\frac{M_u^2}{2}+\beta\cdot\frac{M_u^4}{4}+\gamma\cdot\frac{M_u^6}{6}=M_u\cdot\chi_3-\Omega_M\\ \left.\frac{d\chi(M)}{dM}\right|_{M=0}\cdot\left.\frac{dM(\chi)}{d\chi}\right|_{\chi=0}=1\end{cases} \tag{24}$$

From this system for unknown parameters we obtained:

$$\alpha=\frac{12}{E_s\cdot b\cdot h^3};$$
$$\beta=\frac{12}{M_u^4}\cdot\left(M_u\cdot\chi_3-\Omega_M-\alpha\cdot\frac{M_u^2}{2}\right)-\frac{2}{M_u^3}\cdot(\chi_3-\alpha\cdot M_u); \tag{25}$$
$$\gamma=\frac{3}{M_u^5}\cdot(\chi_3-\alpha\cdot M_u)-\frac{12}{M_u^6}\cdot\left(M_u\cdot\chi_3-\Omega_M-\alpha\cdot\frac{M_u^2}{2}\right)$$

Analysis of this dependence showed that the «moment-curvature» diagram constructed according to (23) almost accurately describes the numerical diagrams. Based on the study, the following conclusions can be drawn.

1. For metal elements, using two-line and three linear diagrams of material deformation, analytical dependencies for the «moment-curvature» diagram were compiled.
2. An approximation of the «moment-curvature» diagram for metal elements is given in the form of polynomials of the third and fifth degrees of the bending moment.
3. Numerical experiments have proven that physical nonlinearity has little effect on the degree of static indetermination, but has a strong effect on the stress-strain state.

Table 2

Results of calculating the ordinates of the deflection diagram at various loading levels

η	$\frac{y}{y_0}$; $\delta_u=1.740$					
	$\delta=0$ $\bar{Q}_A=0.4750$	$\delta=0.2\delta_u$ $\bar{Q}_A=0.4752$	$\delta=0.4\delta_u$ $\bar{Q}_A=0.4754$	$\delta=0.6\delta_u$ $\bar{Q}_A=0.4756$	$\delta=0.8\delta_u$ $\bar{Q}_A=0.4759$	$\delta=\delta_u$ $\bar{Q}_A=0.4761$
0	0	0	0	0	0	0
0.1	0.00284	0.00285	0.00286	0.00287	0.00288	0.00289
0.2	0.00527	0.00529	0.00531	0.00533	0.00535	0.00537
0.3	0.00697	0.00700	0.00703	0.00705	0.00708	0.00711
0.4	0.00775	0.00778	0.00782	0.00785	0.00788	0.00791
0.5	0.00755	0.00758	0.00762	0.00765	0.00768	0.00771
0.6	0.00645	0.00648	0.00651	0.00653	0.00656	0.00659
0.7	0.00467	0.00469	0.00471	0.00474	0.00476	0.00478
0.8	0.00260	0.00261	0.00263	0.00264	0.00266	0.00267
0.9	0.00080	0.00080	0.00081	0.00081	0.00082	0.00083
1.0	0	0	0	0	0	0

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