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DETERMINATION OF THE DYNAMIC RESPONSE AT THE POINT OF SUSPENSION OF THE ROD WHEN FLYING A CERTAIN PART OF IT

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A B S T R A C T

A model and methods for determining the dynamic response at the suspension point of the rod during the flight of its certain part have been constructed. The boundary value problem has been solved. A technique was used that allows elasticity of the solution to this problem. The dynamic reaction at the suspension point of the rod during the flight of its certain part is determined. It is shown that when the rod is rigidly suspended, the dynamic reaction perceived by the suspension point is equal to the weight of the flying part of the rod. Numerical calculations were carried out for practical values of the system parameters. The results obtained will make it possible in each transverse case to determine the dynamic response at the point of suspension of the rod during the flight of its certain part and, as a result, to prevent violations of the tightness of the wellhead.

Keywords: dynamic response; differential equation; rod; spring stiffness; orthogonality.

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1. Introduction

The need to solve this problem arose due to the increasing number of cases of wellhead leakage due to the flight of a certain part of the lifting pipes. Solving the problem may seem simple, but the needs of practice require its solution.

Various cases of determining the dynamic load in the cross sections of bars were considered [1-13]. The question of determining the dynamic load in the cross sections of a suspended rod during the flight of a certain part of it remains open. Therefore, consider a rod with a l_1 length and f cross-section suspended at one end from a spring with stiffness (fig).

At a moment in t time, a certain part of it with a l_o length flies and a dynamic reaction occurs in the remaining part of the rod. The movements of the cross sections of the remaining part of the rod are described by the equation [1].

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} - 2h\frac{\partial u}{\partial t} + g, \qquad t>0,\ 0\le x<l \tag{1}$$

Initial and boundary conditions

$$u\Big|_{t=0} = \frac{\rho g x}{E}\left(l-\frac{x}{2}\right) + \frac{\rho g l_0 x}{E} + \frac{\rho g f(l_0+l)}{C},\ 0\le x<l \tag{2}$$

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \quad 0\le x<l \tag{3}$$

$$Ef\left.\frac{\partial u}{\partial t}\right|_{x=0} = -c\,u\big|_{x=0},\ t>0 \tag{4}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=l} = 0,\ t>0 \tag{5}$$

where u - is the deformation of any cross section of the rod, x - is the coordinate, t - is the time, a - is the speed of sound propagation in the material of the rod, h - is the drag coefficient, g - is the acceleration of gravity, and, accordingly, ρ and E the density and elastic modulus of the material of the rod, l_o - is the length of the flying part of the rod.

The fluid resistance force is negligibly small compared to other forces. Therefore, we will neglect it. Let us reduce equation (1) to a homogeneous form, satisfying the boundary condition (5) by replacing

$$u_1 = u + \frac{\rho g x^2}{2E} - \frac{\rho g l x}{E}, \qquad u_1\big|_{t=0} = u\big|_{t=0} \tag{6}$$

Then the boundary conditions (4) and (5) will relatively have the same form as (4) and (5).

Equation (1) will take the form

$$\frac{\partial^2 u_1}{\partial t^2} = a^2\frac{\partial^2 u_1}{\partial x^2} \tag{7}$$

The solution to equation (7) taking into account the initial (3) and boundary conditions (4) and (5) has the form

$$u_1 = \sum_{i=0}^{\infty} A_i \cos\left(\frac{a\beta_i}{l}t\right)\cos\beta_i\left(1-\frac{x}{l}\right) \tag{8}$$

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where A_i - is a constant, which is determined from the initial condition (2). From expression (8) taking into account the initial condition (6) we obtain

$$\sum_{i=0}^{\infty} A_i \cos\beta_i \left(1-\frac{x}{l}\right) = \frac{\rho g l_0 x}{E} + \frac{\rho g f(l_0+l)}{C} \tag{9}$$

where β_i - are the roots of the transcendental equation, which is obtained from the boundary condition (4)

$$tg(\beta) = -\frac{cl}{Ef\beta} \tag{10}$$

Multiplying both sides of equation (9) by $\cos\beta_i(1-x/l)$ and integrating from 0 to l, taking into account $\cos\beta_i(1-x/l)$ the orthogonality of the functions, we obtain

$$A_i = \frac{4l\rho g l_0(1-\cos\beta_i)}{E\beta_i(2\beta_i+\sin 2\beta_i)} + 2\frac{\rho g f(l_0+l)}{c}\frac{\sin 2\beta_i}{\beta_i(2\beta_i+\sin 2\beta_i)} \tag{11}$$

It should be noted that the proof of the orthogonality of functions is obtained by taking into account $\cos\beta_i(1-x/l)$ the transcendental equation (10). Substituting into expression (8) we get

$$u_1 = \frac{4l\rho g l_0}{E}\sum_{i=0}^{\infty}\frac{(1-\cos\beta_i)}{\beta_i(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{l}t\right)\cos\beta_i\left(1-\frac{x}{l}\right)+ +4\frac{\rho g f(l_0+l)}{C}\sum_{i=0}^{\infty}\frac{\sin\beta_i}{(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{l}t\right)\cos\beta_i\left(1-\frac{x}{l}\right) \tag{12}$$

From expression (6) taking into account formula (12) we obtain

$$u = \frac{4l\rho g l_0}{E}\sum_{i=0}^{\infty}\frac{(1-\cos\beta_i)}{\beta_i(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{l}t\right)\cos\beta_i\left(1-\frac{x}{l}\right)+ +4\frac{\rho g f(l_0+l)}{C}\sum_{i=0}^{\infty}\frac{\sin\beta_i}{(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{l}t\right)\cos\beta_i\left(1-\frac{x}{l}\right)+ +\frac{\rho g l x}{E}-\frac{\rho g x^2}{2E} \tag{13}$$

The dynamic response at the suspension point of the rod can be determined by the formula

$$\sigma = E\left.\frac{\partial u}{\partial x}\right|_{x=0} \tag{14}$$

Then from formula (14) taking into account expression (13) we obtain

$$\sigma = 4\rho g l_0\sum_{i=0}^{\infty}\frac{(1-\cos\beta_i)\sin\beta_i}{\beta_i(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{l}t\right)+ +2\frac{\rho g f E(l_0+l)}{Cl}\sum_{i=0}^{\infty}\frac{\beta_i\sin 2\beta_i\sin\beta_i}{(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{l}t\right)+\rho g l \tag{15}$$

In formula (15), the first two terms are dynamic, and the third is the static component of the reaction that occurs in the upper section of the rod during the flight of a certain part of it.

It should be noted that at $C=\infty$, i.e. when the rod is rigidly suspended, the second term of expression (15) disappears.

The roots of the transcendental equation (10) are determined from the expression

$$tg\beta=\infty \tag{16}$$

Then from expression (16) we obtain $\beta_i = \frac{\pi}{2}(2i+1)$, and

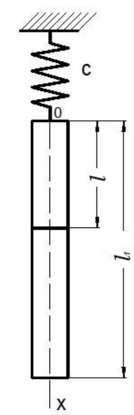

Fig. Calculation scheme

expression (14) will take the form

$$\left.\sigma\right|_{x=0} = 4\rho g l_0\sum_{i=0}^{\infty}\frac{(-1)^i}{\pi(2i+1)}\cos\left(\frac{\pi(2i+1)a}{2l}t\right)+\rho g l \tag{17}$$

From expression (17) it is clear that the maximum value of the dynamic reaction that occurs at the suspension point of the rod during the flight of its part with a l_0 length equal to the weight of the flying part.

This is of more practical importance, since it allows us to determine the probable maximum value of the dynamic response perceived by the gimbal point during the flight of its certain part.

If we $\frac{l_0}{l_0+l}$ denote by ξ then from expression (15) we obtain

$$\left.\frac{\sigma}{\rho g(l+l_0)}\right|_{x=0} = 4\xi\sum_{i=0}^{\infty}\frac{(1-\cos\beta_i)\sin\beta_i}{(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{(1-\xi)l_1}t\right)+ +2\frac{fE}{Cl}\sum_{i=0}^{\infty}\frac{\beta_i\sin 2\beta_i\sin\beta_i}{(2\beta_i+\sin 2\beta_i)}\cos\left(\frac{a\beta_i}{(1-\xi)l_1}t\right)+(1-\xi) \tag{18}$$

Numerical calculations were carried out for the following values of system parameters

$C=10^6,\ 2\cdot 10^8\,\frac{H}{m},\ E=2\cdot 10^{11}\,\frac{H}{m^2},\ f=47.1\cdot 10^{-4}\,m^2,\ \rho=7.8\cdot 10^3\,\frac{kg}{m^3},$
$g=10m/c^2,\ a=5\cdot 10^5\,м/c,\ l_1=3\cdot 10^3,\ 4\cdot 10^3 m,\ f_T=34.4\cdot 10^{-4}\,m^2$
at value $\cos\left(\frac{a\beta_i}{(1-\xi)l}t\right)=1$, ξ=0.1–0.9 values.

Calculations show that with the above values of the $\frac{cl}{Ef}\geq 1000$ system parameters and therefore the roots are not much different from $\frac{(2i+1)}{2}\pi$.

The dynamic reaction force perceived by the suspension point of the rod is determined from the expression

$$Q = f_T\left.\sigma\right|_{x=0} \tag{19}$$

1. So, a solution model has been built to determine the dynamic response at the suspension point of the rod during the flight of its certain part.

2. It is shown that for practical values of the system parameters, the dynamic reaction at the suspension points of the rod during flight of a certain part of it, l_0 the length is equal to the weight of the flying part. For small values c , it is determined by formulas (18) and (19).

Conclusion

An analytical expression is obtained that allows one to determine the dynamic response at the suspension point of the rod during the flight of its certain part.

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