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MATHEMATICAL MODELING OF FILTRATION FLOWS IN A HYDRAULIC FRACTURING CRACK USING THE TECHNOLOGY OF PAIRED HORIZONTAL WELLS

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ABSTRACT

In this paper, a mathematical model of the process of developing a low-permeability reservoir using the technology of paired horizontal channels in the presence of a crack formed during hydraulic fracturing is considered in detail. Hydraulic fracturing is one of the most effective methods of increasing well productivity, in particular, in conditions of complex geological structure of objects. One of the factors of its low effectiveness is an insufficiently high and relevant level of knowledge about the mechanisms of fluid movement in filtration channels formed as a result of injection of liquid under pressure. Assuming that the system of horizontal wells is replaced by one hypothetical well, through which water injection and oil extraction are carried out simultaneously, the problem of the movement of the oil-water boundary between cracks created by hydraulic fracturing in a low-permeability reservoir is considered. A mathematical model is proposed to describe two-phase filtration of liquids (oil and water), and its limits of application for solving real problems of oil field development are indicated. The model is based on the mass balance equations and Darcy's law for each phase, which makes it possible to study the process of injection and selection of reserves in a particular well bore and, consequently, a reservoir section. The results of modeling and analysis of the influence of crack diameters on the distribution of pressure and water saturation are presented. The results obtained make it possible to determine the optimal operating modes of the injection well and predict the water content and flow rate of the producing well using numerical approaches characterized by a low level of errors.

Keywords: hydraulic fracturing; fractures; horizontal wells; low permeability reservoir; filtration; oil.

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Introduction

In the modern oil industry, a promising direction in the development of technologies for the extraction of natural hydrocarbons is the development of low-permeability reservoirs using the technology of paired horizontal wells in the presence of hydraulic fractures. Depletion of light conventional oil reserves, as well as continuous price increases, arouse genuine interest in hard-to-recover oil reserves, which are also contained in low-permeability reservoirs. One of the main methods of developing such deposits is hydraulic fracturing. The theoretical foundations of hydraulic fracturing are presented in the works of S. A. Khristianovich, Yu. P. Zheltov, V. Sh. Shagapov, R. D. Kanevskaya, V. I. Astafiev, L. F. Kemp and others [1-5]. The development of an oil-saturated reservoir using horizontal channels is a relatively new technology in the field of oil production. This method has become widespread in the development of hard-to-recover hydrocarbon reserves. In combination with hydraulic fracturing, this technology is of genuine interest, since a non-trivial geomechanical method of influencing the oil reservoir allows increasing the flow of oil to the well, thereby increasing the flow rate.

Fluid movement in fracturing fractures is characterized by a high level of nonlinearity and requires a detailed study of oil and water flows in various sections of the formation. The use of numerical methods for solving problems in this case is characterized by irrelevant results due to the need to take into account the full dynamics of the process, for example, information about the exact location of cracks in a productive formation. Based on this, an urgent area of research is the development of simplified schemes that could be used to describe fluid flow processes in formations with a low permeability value [6, 7].

The scientific literature describes various approaches to modeling the inflow to horizontal wells with multistage hydraulic fracturing: analytical models, mainly for single-phase filtration. Some take into account the multiphase flow and variable properties of fluids, which increases the accuracy of flow rate estimation; semi-analytical methods, where the flow in cracks is described approximately, and the solution is coupled with the numerical one in the formation,

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which allows for more complex geometry; numerical methods that allow detailed modeling of filtration in all areas, taking into account the heterogeneity of properties, but requiring significant computational costs. Thus, the development of effective simplified filtration models in the formation-crack-well system remains an urgent task to accelerate calculations in the design of hydraulic fracturing.

Models of fluid motion in a porous medium are considered in detail in various literary sources. Note that most of them are based on the consideration of a single-phase fluid, the properties of which do not change over time. So, one of the most relevant models is a system that allows you to predict the value of productivity of wells with a horizontal trunk, on which multi-site hydraulic fracturing was carried out. A feature of this model is the possibility of its use for objects with low permeability and multiphase flows in reservoirs and filtration channels of hydraulic fracturing. In the case of considering single-phase filtration after hydraulic fracturing, the calculation results are characterized by a high level of irrelevance. In addition to analytical models, «semi-analytical methods» have become widespread in fishing practice, in which movement in isolated sections of the reservoir is described using empirical formulas (for example, many scientists use technologies for predicting fluid inflow based on integrated approaches in modeling). With such approaches, the pressure in the hydraulic fracturing filtration channels must be verified with the numerical solution obtained. Taking into account the active implementation of horizontal wells with MHF, especially at facilities in Western Siberia, calculation schemes have become highly relevant, allowing to determine filtration rates in various areas of the formation, including those with complex geometry. It is especially important that the heterogeneity and permeability of deposits are taken into account when conducting modeling. Some researchers have studied in detail the possibilities of non-stationary pressure changes in solving problems of movement of a single-phase fluid in a reservoir with the condition of identifying the influence of a horizontal well with an MHF draining a selected area. In this case, the formation is represented as infinite, and the cracks are located at a certain distance from each other. To simplify the task, a number of geological and physical characteristics of deposits are represented as constant, and the pressure at the point is determined using the Laplace transform. In addition, it is also known about the possibility of simplified formulation of fluid motion problems in a porous medium, taking into account the active operation of fracturing fractures. The simulation uses an iterative process in which the initial task is divided into two particular tasks: studying the movement of reservoir fluids in a porous medium and determining the intensity of the flow of a two-phase liquid in filtration channels.

Research and methods of solving the problem

However, despite the widespread use of hydraulic fracturing, there are still a number of unresolved problems related to optimizing the parameters of the crack and predicting its behavior during the development of the field. One of the promising directions for improving the efficiency of hydraulic fracturing is the use of paired horizontal wells technology. This approach allows you to create extended cracks connecting two horizontal wells, which helps to increase the flow rate and the coefficient of hydrocarbon recovery. However, mathematical modeling of filtration flows in such cracks is a rather difficult task that requires taking into account many factors, including crack geometry, reservoir and fluid properties, as well as the dynamics of changes in filtration characteristics during development. In this regard, the development of mathematical models that adequately describe filtration processes in a hydraulic fracturing crack connecting two horizontal wells is an urgent scientific and practical task. Such models will improve the accuracy of forecasting well flow rates, optimize the parameters of hydraulic fracturing and field development using the technology of paired horizontal wells. The purpose of this work is to develop a mathematical model of filtration flows in a hydraulic fracturing crack connecting two horizontal wells, taking into account changes in its geometric and filtration characteristics during field development.

Two-phase filtration of liquids in porous media plays an important role in many engineering applications, such as oil and gas field development, geothermal energy, etc. The presence of cracks of various diameters can significantly affect the filtration processes and the distribution of fluids in a porous medium. Therefore, taking into account crack diameters when modeling two-phase filtration is an urgent task.

This work is devoted to the mathematical modeling of the filtration process of liquid hydrocarbons in the reservoir using the technology of paired horizontal wells subject to hydraulic fracturing of low-permeability reservoirs (fig.). The development of liquid hydrocarbon deposits with low reservoir properties is currently considered a promising area of oil production, since the reserves contained in them are quite large. This paper presents a theoretical study of the oil filtration process in a porous and permeable medium formed by hydraulic fractures.

Hydraulic fracturing with the formation of extended cracks makes it possible to increase the permeability of the reservoir, as well as increase the area of impact, involving additional hydrocarbon reserves in the development and increase the oil recovery coefficient as a whole.

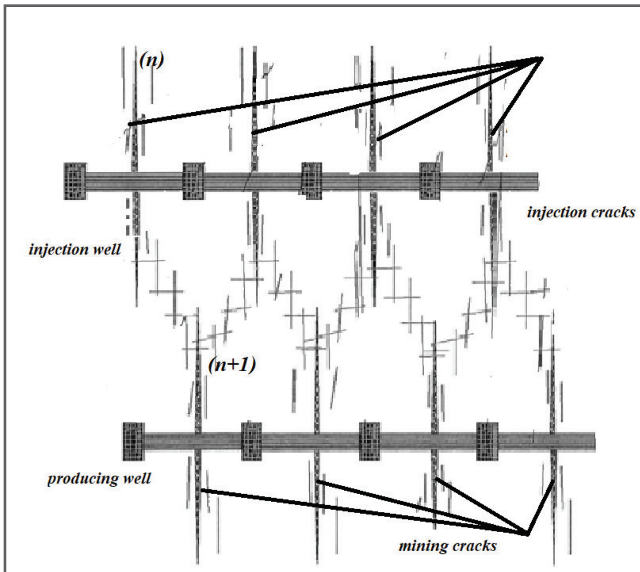

Fig. Diagram of the oil reservoir development process using a system of horizontal wells exposed to hydraulic fracturing

Discussion of the results

Suppose that parallel cracks formed in a low-permeability reservoir after hydraulic fracturing, located perpendicular to a system of two horizontal wells, one of which is injection and the other is producing, and the wells communicate alternately with adjacent hydraulic fracturing cracks. If, for example, there is a crack with a number (n) if it is connected to the injection well, then (n+1) the crack communicates with the producing well. Let the average distances between adjacent cracks be equal to l, and the distance between the wells d. In addition, the distance between the wells is much smaller than the characteristic radial extent of the cracks (d<<R). Cracks communicating with the injection well will be called injection cracks, and those communicating with the production well will be called production cracks. Let water flow through the injection well into the crack, and oil flows into the producing well from the crack, and after a certain period of time, water will partially flow, i.e. flooding will begin. Let's assume that the fluid flow in the crack is radially one-dimensional. Let's write down the mass conservation equation for water, taking into account the inflow from the well and outflow into the porous medium in (n) injection crack as [8, 9]:

$$\frac{\partial\left(m_f\rho_{fw}\right)}{\partial t}+\frac{1}{r}\frac{\partial\left(r\rho_w v_{fw}\right)}{\partial r}=-2\frac{\rho_w v_{pw}}{d_f}\qquad\left(y=0,\, r_c<r<r_f\right),\quad(1)$$

where m_i and ρ_j (i=f,p; j=w,o) here and in the future – porosity and density (the lower indices correspond to the values of the parameters in the crack and its surrounding porous medium); v_i – filtration rate; d_f – the thickness of the crack.

From a certain point on $t=t^{(w)}$ water begins to flow into the mining crack, so from now on ($t>t^{(w)}$) a near zone will form in the crack ($r_c<r<r_w$), where the combined flow of oil and water will take place. Therefore, the equations of conservation of the mass of oil and water in this zone, taking into account the inflow of oil from the porous medium and outflow into the producing well, will be written as:

$$\frac{\partial\left(m_f S_o\rho_o\right)}{\partial t}+\frac{1}{r}\frac{\partial\left(r\rho_o v_{fo}\right)}{\partial r}=0\qquad\left(r_c<r<r_w,\, y=l\right),$$
$$\frac{\partial\left(m_f\left(1-S_o\right)\rho_w\right)}{\partial t}+\frac{1}{r}\frac{\partial\left(r\rho_w v_{fw}\right)}{\partial r}=-2\frac{\rho_w v_{pw}}{d_f}\qquad(r_c<r<r_w,\, y=l),\quad(2)$$

where S_o – oil saturation.

In the far zone, the mass conservation equation has the form:

$$\frac{\partial\left(m_f\rho_o\right)}{\partial t}+\frac{1}{r}\frac{\partial\left(r\rho_o v_{fo}\right)}{\partial r}=2\frac{\rho_o v_{po}}{d_f}\qquad\left(r_w<r<r_f,\, y=l\right).\quad(3)$$

The terms in the right part of equations (1), (2) and (3) express the intensity of outflow and inflow of fluid (water or oil) through its walls into the crack, related to the unit of its area. Therefore, to determine these terms, it is necessary to take into account the filtration process in a porous medium outside the crack. To do this, in turn, assuming that flows in a porous medium occur perpendicular to the crack walls, we write down the mass conservation equation for oil and water [10-15]:

$$\frac{\partial\left(m_p\rho_w\right)}{\partial t}+\frac{\partial\left(\rho_o v_{pw}\right)}{\partial y}=0\qquad\left(0<y<l_{(wo)}\right)$$
$$\frac{\partial\left(m_p\rho_o\right)}{\partial t}+\frac{\partial\left(\rho_o v_{po}\right)}{\partial y}=0\qquad(l_{(wo)}<y<l)\quad(4)$$

Here is the axis Oy it is counted from the walls of the injection crack with the number n. The acceptance of the continuity equations in this form is justified by the fact that the pressure gradients in the radial direction are much smaller than in the direction perpendicular to the crack walls [16-25]. Function $y=l_w(t,r)$ defines the frontal boundary of oil displacement by water.For the described filtration process in a crack and the porous space surrounding it, we will adopt Darcy's law, which for injection, extraction cracks and in a porous medium will be written as:

$$v_{fw}=-\frac{k_f}{\mu_f}\frac{\partial p_f}{\partial r}\qquad(r_c<r<r_f),\quad(5)$$

$$v_{fw}=-\frac{k_f k_w}{\mu_w}\frac{\partial p_f}{\partial r}\quad\left(r_c<r<r_w\right),\ v_{fo}=-\frac{k_f k_o}{\mu_o}\frac{\partial p_f}{\partial r}\quad\left(r_c<r<r_w\right),$$
$$v_{fo}=-\frac{k_f}{\mu_o}\frac{\partial p_f}{\partial r}\qquad(r_w<r<r_f).\quad(6)$$

$$v_{pw}=-\frac{k_p}{\mu_w}\frac{\partial p_p}{\partial r}\frac{\partial p_p}{\partial y}\qquad(0<y<l_{(wo)}),$$
$$v_{po}=-\frac{k_p}{\mu_o}\frac{\partial p_p}{\partial r}\frac{\partial p_p}{\partial y}\qquad(l_{(wo)}<y<l).\quad(7)$$

As initial conditions (t<0) assume that there is a uniform pressure in the formation (including cracks) ($p_f=p_p=p_o$) and up to this point (t<0) there was no injection of water through the cracks ($l_{(wo)}=0$). At a moment in time t=0 water injection begins, increasing the pressure in the injection well near the injection crack to a value of $p_{(n)c}$ ($p_{(n)c}>p_o$). At the same time, a constant pressure value is maintained in the producing crack near the producing well $p_{(n+1)c}$ ($p_{(n+1)c}\le p_o$). We will build a quasi-stationary solution. To do this, it is necessary that the characteristic times for such a system, there were significantly more characteristic times for establishing a stationary regime of pressure distribution in the formation between cracks, that is [26-35]: $t_*\gg t_{pw},t_{po},t_{pi}=\dfrac{l^2}{\chi_{pi}},\ \chi_{pi}=\dfrac{k_p\rho_i c_i}{m_p\mu_i}$, where χ – piezo conductivity coefficient; i=w, o; c_i – the speed of sound for water and oil. Under these assumptions, the solution of equations (4), taking into account (7), has the form:

$$\frac{\partial\left(\rho_w v_{pw}\right)}{\partial y}=0,\rho_w\frac{\partial}{\partial y}\left(-\frac{k_p}{\mu_w}\frac{\partial p_p}{\partial r}\right)=0,\ \frac{\partial p_p}{\partial y}=C_1,\ p_p=C_1 y+C_2,$$

by y=0, $p_p=p_{(n)f}$, $p_{(n)f}=C_2$, $y=l_{(wo)}$, $p_p=p_{(wo)}$, $p_{(wo)}=C_1\cdot l_{(wo)}+C_2$, $C_1=\dfrac{p_{wo}-p_{(n)f}}{l_{wo}}$,

$$p_p=\frac{p_{wo}-p_{(n)f}}{l_{wo}}y+p_{(n)f}\qquad(0<y<l_{wo}),\quad(8)$$

$$y=l_{wo},\, p_p=p_{wo},\ y=l,\ p_p=p_{(n+1)f},\, C_1\left(l-l_{(wo)}\right)=p_{(n+1)f}-p_{(wo)}.$$

$$p_p=\frac{p_{(n+1)f}-p_{(wo)}}{l-l_{(wo)}}\left(y-l_{(wo)}\right)+p_{(wo)}\qquad(l_{(wo)}<y<l).\quad(9)$$

Here $p_{(wo)}$ – the pressure value at the front of oil displacement by water. Let $y=l_{(wo)}$ (t,r), where is the function $l_{(wo)}$ (t,r) is a certain surface, then the equation for the evolution of this surface can be written as:

$$\frac{\partial l_{(wo)}}{\partial t}=\frac{v_{pw}}{m_p}\qquad\left(y=l_{(wo)},\ v_{po}=v_{pw}\right).\quad(10)$$

Taking into account Darcy's law (7) and (9) of (10), we have:

$$\frac{\partial l_{(wo)}}{\partial t}=-\frac{k_p}{m_p\mu_w}\frac{\partial p_p}{\partial y},\ \frac{\partial l_{(wo)}}{\partial t}=-\frac{k_p}{m_p\mu_w}\left(\frac{p_{(wo)}-p_{(n)f}}{l_{(wo)}}\right),$$
$$\frac{\partial l_{(wo)}}{\partial t}=\frac{k_p\left(p_{(n)f}-p_{(wo)}\right)}{m_p\mu_w l_{(wo)}}. \qquad (11)$$

On the other hand:

$$\frac{\partial l_{(wo)}}{\partial t}=-\frac{k_p}{m_p\mu_o}\frac{\partial p_p}{\partial y},\frac{\partial l_{(wo)}}{\partial t}=-\frac{k_p}{m_p\mu_o}\left(\frac{p_{(n+1)f}-p_{(wo)}}{l-l_{(wo)}}\right),$$
$$\frac{\partial l_{(wo)}}{\partial t}=\frac{k_p\left(p_{(wo)}-p_{(n+1)f}\right)}{m_p\mu_o\left(l-l_{(wo)}\right)},\frac{\left(p_{(n)f}-p_{(wo)}\right)}{\mu_w l_{(wo)}}=\frac{\left(p_{(wo)}-p_{(n+1)f}\right)}{\mu_o\left(l-l_{(wo)}\right)},$$
$$\left(p_{(n)f}-p_{(wo)}\right)\left(\mu_o\left(l-l_{(wo)}\right)\right)=\left(p_{(wo)}-p_{(n+1)f}\right)\mu_w l_{(wo)},$$
$$p_{(wo)}\left(\mu_w l_{(wo)}+\mu_o\left(l-l_{(wo)}\right)\right)=p_{(n)f}\mu_o\left(l-l_{(wo)}\right)+p_{(n+1)f}\mu_w l_{(wo)},$$
$$p_{(wo)}=\frac{p_{(n)f}\mu_o\left(l-l_{(wo)}\right)+p_{(n+1)f}\mu_w l_{(wo)}}{\mu_w l_{(wo)}+\mu_o\left(l-l_{(wo)}\right)}. \qquad (12)$$

Substituting expression (12) into the first equation (11), we get:

$$\frac{\partial l_{(wo)}}{\partial t}=\frac{k_p\left(p_{(n)f}-p_{(n+1)f}\right)}{m_p\left(\mu_o\left(l-l_{(wo)}\right)+\mu_w l_{(wo)}\right)}. \qquad (13)$$

From equation (1), taking into account Darcy's law (5), (7) and from equation (8), we obtain the filtration equation for the injection crack with the number (n) as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_f}{\mu_w}\frac{\partial p_{(n)f}}{\partial r}\right)=\alpha^{(p)}\frac{p_{(n)f}-p_{(n+1)f}}{l_{(wo)}+\frac{\mu_o}{\mu_w}\left(l-l_{(wo)}\right)}. \qquad (14)$$

where $\alpha^{(p)}=\frac{2k_p}{d_f k_f}$.

This expression represents the dependence of the pressure change in the crack on the change in the coordinate of the movement of the water front. Similar equations can be obtained for a mining crack from the moment when flooding begins $t=t^{(w)}$.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_w\frac{\partial p_{(n+1)f}}{\partial r}\right)=-\alpha^{(p)}\frac{p_{(n)f}-p_{(n+1)f}}{\left(l-l_{(wo)}\right)+\frac{\mu_w}{\mu_o}l_{(wo)}} \quad (r_c<r<r_w). \qquad (15)$$

Outside the watering zone ($r_w<r<r_f$) the filtration equation for oil will be written similarly to the second equation from (15) by the corresponding substitution $k_o=1$.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_f}{\mu_o}\frac{\partial p_{(n+1)f}}{\partial r}\right)=\alpha^{(p)}\frac{\left(p_{(n+1)f}-p_{(n)f}\right)}{\left(l-l_{(wo)}\right)+\frac{\mu_w}{\mu_o}l_{(wo)}}. \qquad (16)$$

To solve the system of equations (14) and (15) together with equation (13), it is necessary to set boundary conditions for filtration equations in injection and production cracks. As such conditions, we assume that the pressure in the injection crack at $t=0$ suddenly rises to a value $p_{(n)f}=p_{(n)c}$ by $r=r_c$, and for a mining crack, the pressure remains at the value $p_{(n+1)f}=(p_{(n+1)c}\le p_o)$. As a second boundary condition, we take the condition of the absence of a filtration flow at the crack boundary in the form [36-43]: $\frac{\partial p_{(n)f}}{\partial r}=\frac{\partial p_{(n+1)f}}{\partial r}\left(r=r_f\right)$. In addition, we take as the initial condition for the displacement front:

$$l_{(wo)}=0 \quad \left(t=o\right),k_w=S^3,k_o=(1-S)^3.$$
$$k_w=\begin{cases}0 & \left(0\le S\le 0.2\right);\\ \left(S-0.2\right)/0.8)^{3.5} & \left(0.2<S<1\right).\end{cases}$$
$$k_o=\begin{cases}\left(\left(0.85-S\right)/0.85\right)^{3.5} & \left(0<S<0.85\right);\\ 0 & \left(0.85<S<1\right).\end{cases}$$

Let's find an analytical solution to the resulting system of equations for the initial stage (before the start of watering):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_{(n)f}}{\partial r}\right)=\alpha_1\left(p_{(n)f}-p_{(n+1)f}\right)^{x\alpha_2}, \qquad (14^*)$$

where

$$\alpha_1=\frac{\alpha^{(p)}}{l_{(wo)}+\frac{\mu_o}{\mu_w}\left(l-l_{(wo)}\right)},\quad l_{(wo)}=0;$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_{(n+1)f}}{\partial r}\right)=-\alpha_2\left(p_{(n)f}-p_{(n+1)f}\right)^{x\alpha_1}, \qquad (15^*)$$

where

$$\alpha_2=\frac{\alpha^{(p)}}{l-l_{(wo)}+\frac{\mu_w}{\mu_o}l_{(wo)}},\quad l_{(wo)}=0.$$

Let's take the boundary conditions:
$p_{(n)f}=p_o$; $p_{(n+1)f}=p_o$ at $r\to\infty$; $p_{(n)f}=p_{(n)c}$; $p_{(n+1)f}=p_{(n+1)c}$ at $r=r_c$.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\alpha_2 r\frac{\partial p_{(n)f}}{\partial r}\right)=\alpha_1\alpha_2\left(p_{(n)f}-p_{(n+1)f}\right),$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(\alpha_1 r\frac{\partial p_{(n+1)f}}{\partial r}\right)=\alpha_1\alpha_2\left(p_{(n)f}-p_{(n+1)f}\right),$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\alpha_2 p_{(n)f}+\alpha_1 p_{(n+1)f}\right)\right)=0.$$

Let's integrate the resulting equation twice:

$$\alpha_2 p_{(n)f}+\alpha_1 p_{(n+1)f}=\left(\alpha_1+\alpha_2\right)p_o.$$

Let's denote $\Delta p_i=p_i-p_o$, $\Delta p_{(n)}=p_{(n)f}-p_o$, $\Delta p_{(n+1)}=p_{(n+1)f}-p_o$, then $a_2\Delta p_{(n)f}+a_1\Delta p_{(n+1)f}=0$.

Let's write down the equations using Δp_i.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Delta p_{(n)f}}{\partial r}\right)=\alpha_1\left(\Delta p_{(n)f}-\Delta p_{(n+1)f}\right). \qquad (14^{**})$$

Let's substitute $\Delta p_{(n+1)f}$.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Delta p_{(n)f}}{\partial r}\right)=\alpha_1\left(\Delta p_{(n)f}+\frac{\alpha_1}{\alpha_2}\Delta p_{(n+1)f}\right),$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Delta p_{(n)f}}{\partial r}\right)=\left(\alpha_1+\alpha_2\right)\Delta p_{(n)f}.$$

Let's introduce the notation $a_1+a_2=\alpha$ and $R=r\sqrt{D}$, then we get:

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Delta p_{(n)f}}{\partial R}\right)=\Delta p_{(n)f}.$$

The solution to this equation is a certain function:

$$\Delta p_{(n)f}=C_1K_0\left(R\right)+C_2J_0\left(R\right),$$

where $J_o(R)$ – the Bessel function of the zero order of the first kind; $K_o(R)$ – a modified zero-order Bessel function of the second kind (MacDonald function), parameters C_1 and C_2 determined from the boundary conditions:

$$\Delta p_{(n)f} = C_1 K_0(R), \text{ at } R = R_c : \Delta p_{(n)c} = C_1 K_0(R_c),\ C_1 = \frac{\Delta p_{(n)c}}{K_0(R_c)}.$$

Against the background of the obtained solutions, we determine the initial distribution of the boundary $l_{(wo)}$:

$$l_{(wo)}^{j+1} = \frac{k_p(\Delta p_{(n)f} - \Delta p_{(n+1)f})}{m_p\left(\mu_o(l - l_{(wo)}) + \mu_w l_{(wo)}\right)} \cdot \tau + l_{(wo)}$$

For the mass flow of water and oil, taking into account Darcy's law, we write:

$$q^{(n)} = -2\pi d_f r_c \frac{k_f}{\mu_w}\left(\frac{\partial p_{(n)}}{\partial r}\right)_{r_c}$$

The obtained equations with the specified boundary conditions allow us to describe the process of filtration and displacement of oil by water in a system of cracks. The expressions obtained in the work allow us to analyze the filtration process in cracks at the initial stage and determine the initial distribution of the phase interface. By substituting the set pressure values in the injection and production cracks, we obtained more specific expressions for the pressure distribution and the position of the phase boundary, however, to obtain numerical results, it is necessary to set the reservoir parameters. The mass flow rate of liquid through the crack also remains unchanged, since it depends on the pressure gradient on the well wall, and not on the absolute pressure value in the formation [44-45]. Thus, taking into account the pressure in the formation affects only the pressure distribution in the injection crack, but not the initial distribution of the interface and the mass flow of liquid through the crack. The solutions obtained assume that all system parameters (permeability, viscosity of the liquid, etc.) remain constant throughout the considered time period. In real conditions, these parameters may change, which will lead to deviations from the calculated value. The proposed solution makes it possible to determine the pressure distribution in cracks and the evolution of the oil displacement front by water.

Conclusions

1. The use of paired horizontal wells subject to hydraulic fracturing is aimed at increasing the efficiency of oil reservoir development by increasing the impact area, involving additional hydrocarbon reserves in the development.
2. The adopted model allows us to study the development of an oil-bearing reservoir using the technology of paired horizontal channels for the production of hydrocarbons in the form of a single one.
3. A mathematical model of filtration fluid flows in a crack at the initial stage before flooding has been obtained, which can be adapted to the conditions of this deposit.
4. Numerical and analytical solutions have been obtained that make it possible to select optimal parameters and modes for conducting geomechanical impacts on the formation.
5. It should be noted that for the effective development of the field by the proposed technology, a detailed study of the physico-chemical properties of oil, rock and other factors of this field is required.

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