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# IMPROVEMENT OF LOAD-BEARING CAPACITY OF PLUNGER–BUSHING CONTACT PAIRS

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A B S T R A C T

This work investigates the issues of modeling and optimizing the performance of precision pair components of the "bushing–shaft" type operating under complex mechanical, thermal, and tribological loading conditions. Particular attention is devoted to increasing the load-bearing capacity, wear resistance, and durability of precision pairs used in oilfield and pumping equipment. During operation, the interaction between the shaft and the bushing leads to contact pressure, friction forces, non-uniform heating, thermoelastic deformations, and abrasive wear, which significantly affect the stress-strain state and service life of the components. A mathematical model of the precision pair has been developed based on the equations of thermoelasticity, heat conduction theory, contact mechanics, and wear kinetics. The shaft and the bushing are considered isotropic elastic bodies with real machined surfaces, whose roughness profiles are represented by truncated Fourier series. The stress-strain state, temperature field, contact pressure, and wear of the precision pair components were determined using the perturbation method, Kolosov–Muskhelishvili complex potentials, the separation of variables method, and numerical calculation techniques. Unlike existing approaches, the proposed model makes it possible to account for the combined influence of surface microgeometry, temperature field, friction, and wear on the parameters of contact interaction. Special attention is paid to investigating the influence of surface roughness on the distribution of contact pressure, temperature, and wear intensity. Optimization criteria aimed at reducing maximum stresses and temperature, ensuring a more uniform distribution of contact pressure and wear, and improving the operational reliability of the precision pair have been formulated. The optimization problems were reduced to linear programming and least-squares problems and solved numerically using the simplex method and the Gaussian elimination method with pivot selection. The proposed approach also makes it possible to investigate limiting thermal states and fracture processes of bushing elements in the presence of cracks. Stress intensity factors, crack-face interaction, and limiting equilibrium conditions were analyzed, making it possible to determine the permissible operational and design parameters of the precision pair. The obtained results provide a theoretical basis for selecting a rational roughness class and optimizing the microgeometry of precision pair component surfaces in order to improve their load-bearing capacity, wear resistance, thermal stability, and operational reliability.

**Keywords:** contact pair; bushing–shaft; thermoelasticity; contact pressure; abrasive wear; surface roughness; microgeometry optimization; stress–strain state; heat conduction; fracture mechanics.

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## 1. Introduction

The main directions of economic and social development place new demands on science. These demands are associated with the need to improve the quality, reliability, and durability of machinery while reducing material consumption. Meeting these requirements is impossible without reliable methods for the design and calculation of machines and mechanisms. One of the most critical components determining the reliability and service life of machinery and equipment are precision (kinematic) pairs used in oilfield equipment and many transport machines.

The service life of equipment and machinery is largely determined by the performance of precision pair components, their wear resistance, and the stress distribution in the interaction zones.

The increasing rate of oil production requires a significant expansion in the manufacture of oilfield equipment and improvement of its quality, including oil pumps, the demand for which will continue to grow in the future [1-3].

Downhole equipment is considered [1, 2, 4-6] one of the most widespread types of equipment in the oil production industry. In the oil and gas industry, various hydraulic machines and mechanisms are used for the extraction and transportation of oil and gas, including drilling pumps, sucker-rod pumps, and diaphragm pumps. The main components of deep-well pumps, whose design and operating conditions determine the performance of the entire pump, are the plunger-cylinder precision pair and the valve pair [3, 5, 6].

The durability of machinery is of particular importance

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for the oil and gas industry, since most components and assemblies of oilfield equipment operate under extremely severe conditions and are subjected to significant loads, intensive wear, fatigue failure, and corrosion. It is estimated [2] that the annual repair cost of equipment averages about 25% of its initial value. In oil machinery engineering, the primary causes of equipment and machine failure are abrasive wear, fatigue fracture, and corrosion of components [1, 2, 4, 6-8]. The main reason for the insufficient service life of precision pairs such as “shaft-bushing”, drilling pump cylinder liner-piston systems, as well as components of centrifugal downhole pumps, electric drills, and many other oilfield equipment units, is abrasive wear.

Among all types of downhole equipment, more than 90% of operating wells rely on various types of pumping equipment for oil extraction. Therefore, special attention must be paid to pumping equipment in order to improve the durability of oilfield machines and mechanisms. A characteristic feature of such equipment is the presence of cylinders and plungers of different designs performing reciprocating motion. The reciprocating movement is transmitted to the pump plunger [5] from the pumping unit through the sucker-rod string.

Research on stress distribution over rough surfaces has been conducted over the past forty years, and substantial results have already been obtained.

Surface failure of the “bushing-shaft” precision pair during friction is closely related to the contact area and the pressure acting on it. The contact area under load is formed as a result of penetration or deformation of individual surface asperities. The properties of the contact zone are important factors in precision pair calculations. In 1895, Hertz solved the elasticity theory contact problem concerning the compression of ideally smooth bodies with initial line or point contact. Hertz’s results were later widely used in contact area calculations.

Analysis of domestic and foreign studies has shown that existing methods for solving contact problems in elasticity theory for annular layers are mainly applicable separately to thin or thick layers, as well as compressible or incompressible materials. At the same time, these methods generally allow solving only the contact problem itself.

Therefore, one of the urgent scientific problems is the development of more general methods for solving contact problems for annular layers of arbitrary thickness, taking into account friction forces in the contact zone and the temperature field arising due to external friction in the precision pair. The developed methods should make it possible, within a unified framework, not only to solve the contact problem itself but also to comprehensively investigate the stress-strain state of the annular layer.

The tribological and strength reliability of precision pair elements are characterized by several indicators, among which the most important are wear resistance, crack resistance, fatigue strength, and others.

Preventing premature failure of precision pair elements in oilfield equipment, transport machinery, and related systems is a major engineering objective. Therefore, the selection of materials and dimensions for precision pair elements should be based on the above criteria. During the design stage of new movable joints, cases where cracks may occur in individual components (bushing, shaft) must also be considered. In this regard, a limit-state analysis of precision pair elements should be performed to ensure that potential initial cracks, even when located in the most unfavorable positions, will not propagate to catastrophic dimensions or cause failure during the expected service life. The minimum initial crack size should therefore be regarded as a design characteristic of the material.

Scientific and technological progress requires improving the quality of all manufactured products, including materials that determine the reliability and service life of structures, machines, and mechanisms. One of the most important tasks is the prevention of premature failure of such products and, consequently, the extension of their service life.

Based on the above, the calculation of precision pair components gives rise to a number of complex problems, many of which remain unresolved to this day, which motivated the present study.

At the current stage of technological and economic development, optimal structural design occupies an important place as one of the most relevant fields in the mechanics of deformable solids. This is due to the fact that problems of optimal structural design arise in many applied areas, including mechanical engineering, shipbuilding, aerospace engineering, and civil construction. Optimal design makes it possible to reduce material consumption and improve the physical and mechanical characteristics of structures.

The presented review of studies devoted to methods for calculating the stress-strain state of friction precision pairs of the “shaft-bushing” type demonstrates that both domestic and foreign researchers have developed certain approaches for evaluating stress-strain states, temperature fields, and wear. However, the assessment of the stress-strain state, temperature, and wear of friction precision pairs taking into account the real friction surface and material structure defects has not yet been fully resolved.

All of the above-mentioned reviews remain relevant.

## 2. Main part

Calculation of the Temperature of Tribotechnical Joints of Precision Pairs

For the development of measures aimed at increasing the durability of machine units, the study of the thermal stress state of the contact pair (friction unit), as a determining factor of the operating regime, is of great importance.

To control the processes of friction and wear in the contact pair, it is necessary to investigate material failure during friction, caused by contact interaction and accompanied by the combined action of temperature.

One of the causes of stresses and deformations in the bushing of the contact pair is its non-uniform heating.

Let us consider the problem of determining the temperature distribution in the bushing.

The heating of the bushing occurs as a result of friction against its walls during the reciprocating motion of the plunger. Since the frequency of the plunger motion is sufficiently high, the problem is considered under steady-state conditions.

During the operation of the contact pair, a surface heat source acts on the inner surface of the bushing in the contact zone with the shaft, caused by external friction. As a result of this interaction, the temperature of both the bushing and the shaft increases.

The temperature field in the metallic bushing is described by the heat conduction equation [9, 10].

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0 \quad (1)$$

The boundary conditions of the heat conduction problem for the bushing are as follows:

$$\lambda\frac{\partial T}{\partial n}=-Q(\theta) \text{ on the contact area} \quad (2)$$

$$\lambda\frac{\partial T}{\partial n}-\alpha_1(T-T_c)=0 \text{ outside the contact area} \quad (3)$$

$$\lambda\frac{\partial T}{\partial r}+\alpha_2(T-T_c)=0 \text{ on the outer surface of the bushing} \quad (4)$$

Here:

$T(r,\theta)$— temperature function of the metallic bushing;

$\lambda$— thermal conductivity of the bushing material;

$\alpha_1$— heat transfer coefficient from the inner cylindrical surface of the bushing (with ambient temperature $T_c$);

$n$— normal to the inner contour of the bushing;

$\alpha_2$— heat transfer coefficient from the outer cylindrical surface;

$Q(\theta)$— intensity of the surface heat source acting on the bushing.

The thermal conductivity coefficients of the material in the axial, circumferential, and radial directions are assumed to be identical and independent of coordinates and temperature.

For the potential of the surface heat source in the friction zone, we have:

$$Q(\theta)=\alpha_m f\rho(\theta)V \quad (5)$$

$f$— coefficient of friction of the contact pair "bushing–shaft", $V$— average relative velocity of the shaft with respect to the bushing, $\rho(\theta)$— contact pressure on the friction surface, $\alpha_{m.n.}$— heat flux partition coefficient for the bushing.

We assume that the inner contour of the bushing is close to circular. As is known, the real surface of the bushing is never absolutely smooth, but always has irregularities that are an inevitable consequence of the technological processing.

Let us refer the bushing to a polar coordinate system $r$, $\theta$, choosing the origin at the center of concentric circles $L_0$ and $L$ with radius $R_0$ and $R$.

To avoid ambiguity in notation, $L_0$ and $L$ denote the inner and outer circular contours, respectively, whereas $\lambda$ is used only for the thermal conductivity coefficient of the bushing material.

Let us consider a certain realization of the rough inner surface of the bushing. Despite the extremely small size of the irregularities, they have a significant influence on the temperature field in points close to the surface, as well as on various operational properties of the tribological contact [6, 11, 12].

Let us represent the boundary of the inner contour $L'$ in the form (1).

The Fourier coefficients used in the representation of the inner contour have a clear physical meaning in the proposed model. They characterize the deviation of the real machined surface of the bushing from the ideal circular contour. Low-order coefficients describe large-scale deviations of the surface profile, whereas higher-order coefficients correspond to smaller irregularities and roughness elements. Variation of these coefficients changes the local curvature of the contact surface and therefore affects the distribution of contact pressure, temperature field, thermal stresses, and wear intensity. Thus, the Fourier coefficients can be considered as control parameters for optimizing the microgeometry of the friction surface.

Let us introduce the excess temperature into consideration

$$T'=T-T_c \quad (6)$$

The temperature function in the bushing is sought in the form of an expansion with respect to a small parameter

$$T=T^{(0)}+\varepsilon T^{(1)}+\cdots \quad (7)$$

in which terms containing the small parameter ε of order higher than the first are neglected.

Here, the functions $T^{(0)}$ and $T^{(1)}$ represent the temperatures of the bushing in the zero and first approximations.

The values of the components of the temperature function at $r=\rho(\theta)$ are found by expanding the temperature function in a series in the neighborhood of $r=R$.

The boundary conditions of the heat conduction problem with accuracy up to first-order small quantities take the following form: for the zero approximation

$$r=R \quad \lambda\frac{\partial t^{(0)}}{\partial r}=-Q_b^0(\theta) \text{ at the contact site} \quad (8)$$

$$r=R \quad \lambda\frac{\partial t^{(0)}}{\partial r}-\alpha_1 t^{(0)}=0 \text{ outside the contact pad} \quad (9)$$

$$r=R_0 \quad \lambda\frac{\partial t^{(0)}}{\partial r}+\alpha_2 t^{(1)}=0$$

for the first approximation

$$r=R \quad \frac{\partial t^{(1)}}{\partial r}=-\frac{Q_h^{(1)}(\theta)}{\lambda}-\frac{\partial^2 t^{(0)}}{\partial r^2}H(\theta) \text{ at the contact site} \quad (10)$$

$$r=R \quad \lambda\frac{\partial t^{(1)}}{\partial r}-\alpha_1 t^{(1)}=\left[\alpha_1\frac{\partial t^{(0)}}{\partial r}-\lambda\frac{\partial^2 t^{(0)}}{\partial r^2}\right]H(\theta)$$

outside the contact pad

$$r=R_0 \quad \lambda\frac{\partial t^{(1)}}{\partial r}+\alpha_1 t^{(1)}=0 \quad (11)$$

In each approximation, the solution of the heat conduction differential equation is sought using the method of separation of variables [13], assuming

$$t^{(0)}=\Phi^{(0)}(\theta)f^{(0)}(r);\quad t^{(1)}=\Phi^{(1)}(\theta)f^{(1)}(r) \quad (12)$$

where, for the temperature to be single-valued, the functions $\Phi^{(0)}(\theta)$ and $\Phi^{(1)}(\theta)$ must be periodic.

It can be assumed that the functions $\Phi^{(0)}(\theta)$ and $\Phi^{(1)}(\theta)$ can be represented as Fourier series.

For the zero approximation, we obtain

$$\begin{aligned} t^{(0)}=C_{10}+C_{20}\ln r+\sum_{k=1}^{\infty}(C_{10}^k r^k+C_{20}^k r^{-k})\cos k\theta+ \\ +\sum_{k=1}^{\infty}(A_{10}^k r^k+A_{20}^k r^{-k})\sin k\theta \end{aligned} \quad (13)$$

To determine the constants $C_{10}$, $C_{20}$, $C_{10}^{(k)}$, $C_{20}^{(k)}$, $A_{10}^{(k)}$, $A_{20}^{(k)}$, the boundary conditions of the problem in the zero approximation were used.

After determining the function $t^{(0)}$, we find the right-hand side of the boundary condition (10) at $r=R$.

Let us represent it in the following form

$$-\frac{Q^{(1)}(\theta)}{\lambda}-\frac{\partial^2 t^{(0)}}{\partial r^2}H(\theta)\sum_{k=-\infty}^{\infty}q_k^{(1)}e^{ik\theta} \quad (14)$$

$$\alpha_1\frac{\partial t^{(0)}}{\partial r}-\lambda\frac{\partial^2 t^{(0)}}{\partial r^2}H(\theta)=\sum_{k=-\infty}^{\infty}d_k^{(1)}e^{ik\theta}$$

The solution of the problem in the first approximation has the following form

$$t^{(1)} = C_{11} + C_{21}\ln r + \sum_{k=1}^{\infty}(C_{11}^{k} r^{k} + C_{21}^{k} r^{-k})\cos k\theta + \\ + \sum_{k=1}^{\infty}(A_{11}^{k} r^{k} + A_{21}^{k} r^{-k})\sin k\theta \quad (15)$$

To determine the constants $C_{11}$, $C_{21}$, $C_{11}^{(k)}$, $C_{21}^{(k)}$, $A_{11}^{(k)}$, $A_{21}^{(k)}$, we use the boundary conditions of the problem in the first approximation.

Based on the obtained formulas, with accuracy up to first-order quantities with respect to ε, for the excess temperature of the inner surface of the bushing of the contact pair, we obtain

$$t_{*} = t/_{r=\rho} = t^{(0)}/_{r=R} + \varepsilon\left[\frac{\partial t^{(0)}}{\partial r} + t^{(1)}\right]/_{r=R} \quad (16)$$

Consequently, for each previously known profile of the machined bushing surface, the thermal state of the bushing in the precision pair can be investigated using relations (13), (15), and (16). Analysis of the causes of irregularities in the profile of the machined bushing surface leads [6, 12, 14] to a roughness model consisting of two components: deterministic and random, the latter being described by a normally distributed random function. As is well known [12], it is more appropriate to perform a statistical description of the inner surface of the bushing by considering the function $H(\theta)$in different cross-sections as realizations of a random variable.

**Calculation of thermal stresses in the bushing of tribotechnical conjugations of precision pairs**

Knowing the temperature distribution in the bushing of the precision pair, it is possible to determine the thermal stresses. Let us consider an arbitrary realization of the rough inner surface of the bushing. It is assumed that the plane strain conditions are satisfied. The boundary conditions for the thermal stress problem on the inner and outer contours of the bushing will have the following form:

$$v_r^T = 0; \quad v_\theta^T = 0 \quad \text{at} \quad r = R_0$$
$$\sigma_n^T = 0; \quad \tau_{nt}^T = 0 \quad \text{at} \quad r = \rho(\theta) \quad (17)$$

Here it is assumed that the outer surface of the bushing is fitted into a rigid casing. In what follows, for simplicity, the superscript will be omitted. Repeating the procedure of the method described in §1.2, we obtain the boundary conditions of the thermal stress problem for each approximation.

For the zero-order approximation:

$$\sigma_n^{(0)} = 0; \quad \tau_\theta^{(0)} = 0 \quad \text{at} \quad r = R$$
$$v_r^{(0)} = 0; \quad v_\theta^{(0)} = 0 \quad \text{at} \quad r = R_0 \quad (18)$$

as a first approximation

$$\sigma_r^{(1)} = N; \quad \tau_{r\theta}^{(1)} = T \quad \text{at} \quad r = R$$
$$\sigma_r^{(1)} = N; \quad \tau_{r\theta}^{(1)} = T \quad \text{at} \quad r = R \quad (19)$$

Here, the functions ***N*** and ***T*** are determined by formulas (14). To formulate the thermal stress problem in each approximation, we use the thermoelastic displacement potential [15]. In the problem under consideration, the thermoelastic displacement potential in the zero-order approximation is determined by the differential equation:

$$\Delta\Phi^{(0)} = \frac{1+\mu}{1-\mu}\alpha t^{(0)} \quad (20)$$

The temperature function $t^{(0)} = (r,\theta)$ is taken in the form of a Fourier series (see formula (13)); Δ – is the Laplace operator; $\mu$ – is Poisson's ratio of the bushing material; and $\alpha$ is the coefficient of linear thermal expansion.

The solution of equation (20) for the thermoelastic potential is sought in the following form:

$$\Delta\Phi^{(0)} = \sum_{n=0}^{\infty}[\Phi_n^{(0)}\cos n\theta + \Phi_n^{*(0)}\sin n\theta] \quad (21)$$

For the functions $\Phi_n^{(0)}(r)$ and $\Phi_n^{*(0)}(r)$ we obtain

$$\frac{d^2\Phi_n^{(0)}}{dr^2} + \frac{1}{r}\frac{d\Phi_n^{(0)}}{dr} - \frac{n^2}{r^2}\Phi_n^{(0)} = \frac{1+\mu}{1-\mu}\alpha F_n^{(0)} \quad (22)$$
$$\frac{d^2\Phi_n^{*(0)}}{dr^2} + \frac{1}{r}\frac{d\Phi_n^{*(0)}}{dr} - \frac{n^2}{r^2}\Phi_n^{*(0)} = \frac{1+\mu}{1-\mu}\alpha F_n^{*(0)}$$

Particular solutions of equations (22) are sought using the method of variation of constants

$$\Phi_0^{(0)} = \beta\left[-\ln r\int_R^r \rho F_0^{(0)}(\rho)d\rho + \int_r^{R_0}\rho F_0^{(0)}(\rho)\ln\rho d\rho\right] \quad (23)$$

$$\Phi_n^{(0)} = -\beta\cdot\frac{1}{2n}\left[r^n\int_r^{R_0}F_n^{(0)}(\rho)\rho^{1-n}d\rho + r^{-n}\int_R^r \rho F_n^{(0)}(\rho)\rho^{1+n}d\rho\right]$$

$$\Phi_n^{*(0)} = -\beta\cdot\frac{1}{2n}\left[r^n\int_r^{R_0}F_n^{*(0)}(\rho)\rho^{1-n}d\rho + r^{-n}\int_R^r \rho F_n^{*(0)}(\rho)\rho^{1+n}d\rho\right]$$

where $\beta = \dfrac{1+\mu}{1-\mu}$

Using formulas [15]:

$$\overline{\sigma_r} = -2G\left(\frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}\right); \quad \overline{\sigma_\theta} = -2G\frac{\partial^2\Phi}{\partial r^2};$$
$$\tau_{r\theta} = 2G\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right) \quad (24)$$

and relations (21), (23),we calculate the corresponding stresses

$$\overline{\sigma_r^{(0)}}, \quad \overline{\sigma_\theta^{(0)}}, \quad \overline{\tau_{r\theta}^{(0)}}$$

$$\overline{\sigma_r^{(0)}} = -2G\left\{\frac{1}{r}\sum_{n=0}^{\infty}\left[\frac{\partial\Phi_n^{(0)}}{\partial r}\cos n\theta + \frac{\partial\Phi_n^{*(0)}}{\partial r}\sin n\theta\right] + \right.$$
$$\left. + \frac{1}{r^2}\sum_{n=0}^{\infty}(-n^2)\left[\Phi_n^{(0)}\cos n\theta + \Phi_n^{*(0)}\sin n\theta\right]\right\} \quad (25)$$

$$\overline{\sigma_\theta^{(0)}} = -2G\sum_{n=0}^{\infty}\left[\frac{\partial^2\Phi_n^{(0)}}{\partial r^2}\cos n\theta + \frac{\partial^2\Phi_n^{*(0)}}{\partial r^2}\sin n\theta\right]$$

$$\overline{\tau_{r\theta}^{(0)}} = 2G\left\{-\frac{1}{r^2}\sum_{n=0}^{\infty}n\left[\Phi_n^{*(0)}\cos n\theta - \Phi_n^{(0)}\sin n\theta\right] + \right.$$
$$\left. + \frac{1}{r}\sum_{n=0}^{\infty}n\left[\frac{\partial\Phi_n^{*(0)}}{\partial r}\cos n\theta + \frac{\partial\Phi_n^{(0)}}{\partial r}\sin n\theta\right]\right\}$$

$$\frac{\partial\Phi_n^{(0)}}{\partial r} = -\frac{\beta}{2}\left[r^{n-1}\int_r^{R_0}F_n^{(0)}(\rho)\rho^{1-n}d\rho - r^{-n-1}\int_R^r F_n^{(0)}(\rho)\rho^{1+n}d\rho\right]$$

$$\frac{\partial^2\Phi_n^{(0)}}{\partial r^2} = -\frac{\beta}{2}\left[\begin{array}{l}(n-1)r^{n-2}\int_r^{R_0}F_n^{(0)}(\rho)\rho^{1-n}d\rho - 2F_n^{(0)} + \\ +(n+1)r^{-n-2}\int_R^r F_n^{(0)}(\rho)\rho^{1+n}d\rho\end{array}\right]$$

$$\frac{\partial^2\Phi_n^{*(0)}}{\partial r} = -\frac{\beta}{2}\left[\begin{array}{l}r^{n-1}\int_r^{R_0}F_n^{*(0)}(\rho)\rho^{1-n}d\rho - \\ -r^{-n-1}\int_R^r F_n^{*(0)}(\rho)\rho^{1+n}d\rho\end{array}\right] \quad (26)$$

$$\frac{\partial^2\Phi_n^{*(0)}}{\partial r^2} = -\frac{\beta}{2}\left[\begin{array}{l}(n-1)r^{n-2}\int_r^{R_0}F_n^{*(0)}(\rho)\rho^{1-n}d\rho - 2F_n^{*(0)} + \\ +(n+1)r^{-n-2}\int_R^r F_n^{*(0)}(\rho)\rho^{1+n}d\rho\end{array}\right]$$

According to formulas [15]:

$$\overline{v_r^{(0)}} = \frac{\partial \Phi^{(0)}}{\partial r}; \quad \overline{v_\theta^{(0)}} = \frac{\partial \Phi^{(0)}}{\partial \theta} \tag{27}$$

we calculate the corresponding displacements $\overline{v_r^{(0)}}$ and $\overline{v_\theta^{(0)}}$

$$\overline{v_r^{(0)}} = \sum_{n=0}^{\infty}\left[\frac{\partial \Phi_n^{*(0)}}{\partial r}\cos n\theta + \frac{\partial \Phi_n^{*(0)}}{\partial r}\sin n\theta\right] \tag{28}$$

$$\overline{v_\theta^{(0)}} = \frac{1}{r}\sum_{n=0}^{\infty}\left[(-n)\left\{\Phi_n^{(0)}\sin n\theta + \Phi_n^{*(0)}\cos n\theta\right\}\right]$$

The obtained stresses (25), (26) and displacements (28) do not satisfy the boundary conditions (28) of the thermoelastic state. Therefore, it is necessary to determine the second stress-strain state $\overline{\overline{\sigma_r^{(0)}}}$, $\overline{\overline{\sigma_\theta^{(0)}}}$, $\overline{\overline{\tau_{r\theta}^{(0)}}}$, $\overline{\overline{v_r^{(0)}}}$, $\overline{\overline{v_\theta^{(0)}}}$.

The complex potentials $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$, describing the stress-strain state $\overline{\sigma_r^{(0)}}$, $\overline{\sigma_\theta^{(0)}}$, $\overline{\tau_{r\theta}^{(0)}}$, $\overline{v_r^{(0)}}$, $\overline{v_\theta^{(0)}}$ are sought in the following form [16]:

$$\Phi^{(0)}(z) = \sum_{k=-\infty}^{\infty} a_k z^k; \quad \Psi^{(0)}(z) = \sum_{k=-\infty}^{\infty} a'_k z^k \tag{29}$$

Satisfying the boundary conditions $\overline{\overline{\sigma_r^{(0)}}} = -\overline{\sigma_r^{(0)}}$, $\overline{\overline{\tau_{r\theta}^{(0)}}} = -\overline{\tau_{r\theta}^{(0)}}$ and $\overline{\overline{v_r^{(0)}}} = -\overline{v_r^{(0)}}$, $\overline{\overline{v_\theta^{(0)}}} = -\overline{v_\theta^{(0)}}$ at $r = R$ and $r = R_0$, respectively, we obtain after certain transformations and comparison of terms.

$$a_0 + \overline{a_0} - a'_{-2}R_0^{-2} = A'_0 \tag{30}$$

$$a_0 - \backslash ae\overline{a_0} - a'_{-2}R_0^{-2} = A''_0$$

$$(1-k)a_k R^k + \overline{a_{-k}}R^{-k} - a'_{k-2}R^{k-2} = A'_k$$

$$(1-k)a_k R_0^k - \backslash ae\overline{a_{-k}}R_0^{-k} - a'_{k-2}R_0^{k-2} = A''_k$$

Here

$$-\left(\overline{\sigma_r^{(0)}} - i\overline{\tau_{r\theta}^{(0)}}\right) = \sum_{k=-\infty}^{\infty} A'_k e^{ik\theta} \quad \text{at} \quad r = R \tag{31}$$

$$-\left(\overline{v_r^{(0)}} - i\overline{v_{r\theta}^{(0)}}\right) = \sum_{k=-\infty}^{\infty} A''_k e^{ik\theta} \quad \text{at} \quad r = R_0$$

Using formulas (25)-(26), (29), the thermoelastic stresses in the bushing in the zero approximation are determined in the following form

$$\sigma_r^{(0)} = \overline{\sigma_r^{(0)}} + \overline{\overline{\sigma_r^{(0)}}}; \quad \sigma_\theta^{(0)} = \overline{\sigma_\theta^{(0)}} + \overline{\overline{\sigma_\theta^{(0)}}}; \quad \tau_{r\theta}^{(0)} = \overline{\tau_{r\theta}^{(0)}} + \overline{\overline{\tau_{r\theta}^{(0)}}} \tag{32}$$

For the displacements of the bushing caused by thermoelastic loading in the zero approximation,we obtain

$$v_r^{(0)} = \overline{v_r^{(0)}} + \overline{\overline{v_r^{(0)}}}; \quad v_\theta^{(0)} = \overline{v_\theta^{(0)}} + \overline{\overline{v_\theta^{(0)}}} \tag{33}$$

The thermoelastic displacement potential in the first approximation is determined by the following equation

$$\Delta\Phi^{(1)} = \beta t^{(1)} \tag{34}$$

The temperature function is represented in the form of a Fourier $t^{(1)}(r,\theta)$ series (see formula (35).

The solution of equation (34) is sought in the following form

$$\Delta\Phi^{(1)} = \sum_{n=0}^{\infty}\left[\Phi_n^{(1)}\cos\theta + \Phi_n^{*(1)}\sin\theta\right] \tag{35}$$

For the functions $\Delta\Phi_n^{(1)}(r)$ and $\Delta\Phi_n^{*(1)}(r)$ we obtain

$$\frac{d^2\Phi_n^{(1)}}{dr^2} + \frac{1}{r}\frac{d\Phi_n^{(1)}}{dr} - \frac{n^2}{r^2}\Phi_n^{(1)} = \beta F_n^{(1)} \tag{36}$$

$$\frac{d^2\Phi_n^{*(1)}}{dr^2} + \frac{1}{r}\frac{d\Phi_n^{*(1)}}{dr} - \frac{n^2}{r^2}\Phi_n^{*(1)} = \beta F_n^{*(1)}$$

Using the method of variation of constants,we obtain particular solutions of equations (35)

$$\Delta\Phi_n^{(1)} = -\frac{\beta}{2n}\left[r^n\int_r^{R_0} F_n^{(1)}(\rho)\rho^{1-n}d\rho + r^{-n}\int_r^{R_0} F_n^{(1)}(\rho)\rho^{1+n}d\rho\right] \tag{37}$$

$$\Delta\Phi_n^{*(1)} = -\frac{\beta}{2n}\left[r^n\int_r^{R_0} F_n^{*(1)}(\rho)\rho^{1-n}d\rho + r^{-n}\int_r^{R_0} F_n^{*(1)}(\rho)\rho^{1+n}d\rho\right]$$

Based on formulas (24), (27) and relations (35), (37),we calculate the corresponding stresses $\overline{\sigma_r^{(1)}}$, $\overline{\sigma_\theta^{(1)}}$, $\overline{\tau_{r\theta}^{(1)}}$, $\overline{v_r^{(1)}}$ and $\overline{v_\theta^{(1)}}$ displacements $\overline{v_r^{(1)}}$, $\overline{v_\theta^{(1)}}$.

The obtained stresses and displacements do not satisfy the boundary conditions (19) of the first approximation of the thermoelastic state. Therefore,it is necessary to determine the second stress-strain state $\overline{\overline{\sigma_r^{(1)}}}$, $\overline{\overline{\sigma_\theta^{(1)}}}$, $\overline{\overline{\tau_{r\theta}^{(1)}}}$, $\overline{\overline{v_r^{(1)}}}$ and $\overline{\overline{v_\theta^{(1)}}}$.

The boundary conditions (19) for their determination take the following form

$$\overline{\overline{\sigma_r^{(1)}}} = N - \overline{\sigma_r^{(1)}}; \quad \overline{\overline{\tau_{r\theta}^{(1)}}} = T - \overline{\tau_{r\theta}^{(1)}} \quad \text{at} \quad r = R \tag{38}$$

$$\overline{\overline{v_r^{(1)}}} = -\overline{v_r^{(1)}}; \quad \overline{\overline{v_{r\theta}^{(1)}}} = -\overline{v_{r\theta}^{(1)}} \quad \text{at} \quad r = R_0$$

The complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$, describing the stress-strain state $\overline{\overline{\sigma_r^{(1)}}}$, $\overline{\overline{\sigma_\theta^{(1)}}}$, $\overline{\overline{\tau_{r\theta}^{(1)}}}$, $\overline{\overline{v_r^{(1)}}}$ and $\overline{\overline{v_\theta^{(1)}}}$, are sought in a form analogous to (29), i.e."

$$\Phi^{(1)}(z) = \sum_{k=-\infty}^{\infty} a_k^{(1)} z^k; \quad \Psi^{(1)}(z) = \sum_{k=-\infty}^{\infty} a_k'^{(1)} z^k \tag{39}$$

Satisfying the boundary conditions (38) by functions (39), after certain transformations we obtain a linear algebraic system with respect to $a_k^{(1)}$ and $a_k'^{(1)}$ of type (30), in which it should be assumed

$$-\left(\overline{\sigma_r^{(1)}} - i\overline{\tau_{r\theta}^{(1)}}\right) = \sum_{k=-\infty}^{\infty} A_k'^{(1)} e^{ik\theta} \quad \text{at} \quad r = R \tag{40}$$

$$-\left(\overline{v_r^{(1)}} - i\overline{v_{r\theta}^{(1)}}\right) = \sum_{k=-\infty}^{\infty} A_k''^{(1)} e^{ik\theta} \quad \text{at} \quad r = R_0$$

The solution of the system of type (1.60) does not present any difficulties.

After determining the coefficients $a_k^{(1)}$ and $a_k'^{(1)}$ using the Kolosov-Muskhelishvili formulas [16], we obtain the stresses $\overline{\overline{\sigma_r^{(1)}}}$, $\overline{\overline{\sigma_\theta^{(1)}}}$, $\overline{\overline{\tau_{r\theta}^{(1)}}}$ and the displacements $\overline{\overline{v_r^{(1)}}}$, $\overline{\overline{v_\theta^{(1)}}}$.

For stresses and displacements in the first approximation we have

$$\sigma_r^{(1)} = \overline{\sigma_r^{(1)}} + \overline{\overline{\sigma_r^{(1)}}}; \quad \sigma_\theta^{(1)} = \overline{\sigma_\theta^{(1)}} + \overline{\overline{\sigma_\theta^{(1)}}}; \quad \tau_{r\theta}^{(1)} = \overline{\tau_{r\theta}^{(1)}} + \overline{\overline{\tau_{r\theta}^{(1)}}};$$

$$v_r^{(1)} = \overline{v_r^{(1)}} + \overline{\overline{v_r^{(1)}}}; \quad v_\theta^{(1)} = \overline{v_\theta^{(1)}} + \overline{\overline{v_\theta^{(1)}}}; \tag{41}$$

For compactness, only the main analytical relations required for determining the thermal stresses and displacement components are presented below. Intermediate algebraic transformations and auxiliary coefficients are omitted, since they do not change the final result and significantly increase the volume of the formulas. This form of presentation makes the obtained relations more convenient for further numerical implementation and engineering analysis.

Finally, for temperature stresses and displacements in the bushing we have the following relations.

In the following relations, the displacement components are denoted uniformly as $v_r$ and $v_\theta$, where $v_r$ is the radial displacement component and $v_\theta$ is the circumferential displacement component. The same notation is used throughout the paper.

$$\sigma_r^{(T)} = \sigma_r^{(0)} + \varepsilon\sigma_r^{(1)}; \quad \sigma_\theta^{(T)} = \sigma_\theta^{(0)} + \varepsilon\sigma_\theta^{(1)} \tag{42}$$

$$\tau_{r\theta}^{(T)} = \tau_{r\theta}^{(0)} + \varepsilon\tau_{r\theta}^{(1)}; \quad \upsilon_r^{(T)} = \upsilon_r^{(0)} + \varepsilon\upsilon_r^{(1)}; \quad \upsilon_\theta^{(T)} = \upsilon_\theta^{(0)} + \varepsilon\upsilon_\theta^{(1)}$$

In these formulas, the quantities $\sigma_r^{(0)}, \sigma_\theta^{(0)}, \tau_{r\theta}^{(0)}$, are determined by relations (25), (26), (29), (32), while the quantities $\sigma_r^{(1)}, \sigma_\theta^{(1)}, \tau_{r\theta}^{(1)}$ are determined respectively by formulas (41), (24), (35), (37), (39).

**Minimization of the stress state in tribotechnical joints of precision components**

Based on the results obtained in the previous sections, the stress state of the bushing-type component of the precision pair is minimized using two approaches:

a) application of the criterion for minimizing the maximum normal tangential stress on the contact surface;

b) application of the principle of equal strength.

**Minimization of the maximum stress in the bushing of a precision pair**

For the first approach, the normal tangential stress on the contact surface $r = \rho(\theta)$ of the bushing is expressed as follows:

$$\sigma_* = \sigma^{(0)}_{\theta/_{r=R}} + \varepsilon\left[H(\theta)\frac{\partial\sigma_\theta^{(0)}}{\partial r} + \sigma_\theta^{(1)}\right]/_{r=R} \tag{43}$$

Here

$$\sigma^{(0)}_{\theta/_{r=R}} = \sigma^{(0)}_{\theta\rho/_{r=R}} + \overline{\sigma^{(0)}_{\theta/_{r=R}}} + \overline{\overline{\sigma^{(0)}_{\theta/_{r=R}}}} \tag{44}$$

$$\frac{\partial\sigma_\theta^{(0)}}{\partial r_{/r=R}} = \frac{\partial\sigma_{\theta\rho}^{(0)}}{\partial r_{/r=R}} + \frac{\partial\overline{\sigma_\theta^{(0)}}}{\partial r_{/r=R}} + \frac{\partial\overline{\overline{\sigma_\theta^{(0)}}}}{\partial r_{/r=R}}$$

$$\sigma^{(1)}_{\theta/_{r=R}} = \sigma^{(1)}_{\theta\rho/_{r=R}} + \overline{\sigma^{(1)}_{\theta/_{r=R}}} + \overline{\overline{\sigma^{(1)}_{\theta/_{r=R}}}}$$

The quantities $\sigma^{(0)}_{\theta\rho/_{r=R'}}, \overline{\sigma^{(0)}_{\theta/_{r=R}}}$ and $\overline{\overline{\sigma^{(0)}_{\theta/_{r=R}}}}$ are determined from formulas (25), (26), (29) and (32) at $r = R(\rho = 1)$. Accordingly, the functions $\sigma^{(1)}_{\theta\rho/_{r=R'}}, \overline{\sigma^{(1)}_{\theta/_{r=R}}}, \overline{\overline{\sigma^{(1)}_{\theta\rho/_{r=R}}}}$ are determined from relations (41), (24), (35), (39) at $r = R\,(\rho = 1)$. For the function $\sigma_*(\theta)$, we determine its maximum value on the contact surface of the bushing

$$\sigma_{*\max} = \sigma_\theta^{(0)}(\theta_*) + \varepsilon\left[H(\theta_*)\frac{\partial\sigma_\theta^{(0)}(\theta_*)}{\partial r} + \sigma_\theta^{(1)}(\theta_*)\right] \tag{45}$$

Here, the value $\theta_*$ is the root of the equation

$$\frac{d\sigma_*}{d\theta} = 0 \tag{46}$$

The coefficients $a_k^0$ and $b_k^0$ of the sought function $H(\theta)$ must be chosen in such a way that the minimization of $\sigma_{*\max}$ is ensured (minimax criterion).

In the formulated optimization problem, the stress $\sigma_{*\max}$ (the performance index) depends linearly on the unknown coefficients. Therefore, the optimization problem is reduced to a linear programming problem.

In this case, the following constraint must be satisfied

$$\sigma_{*\max} \le [\sigma] \tag{47}$$

where [σ]represents the allowable stress for the bushing material, determined experimentally.

Knowing which type of mechanical processing is used in the manufacturing of the bushing-type component, additional constraints must be imposed on the main characteristics of the surface microgeometry.

As is known [17, 18], roughness characteristics are evaluated mainly from profilograms within a standardized base length. A system is adopted in which the heights of irregularities are measured relative to the mean line. The mean profile line is a baseline having the shape of the nominal profile and dividing the measured profile in such a way that the sum of the squares of the deviations of profile points located above and below the mean line is minimal.

In addition, through the peak of the highest irregularity and the deepest valley, lines of peaks and valleys are drawn parallel to the mean line.

The characteristics of the profile microgeometry are as follows:

$$R_a = \frac{1}{n}\sum_{i=1}^{n}\left|H(\theta_i)\right|$$

$R_a$ — the arithmetic mean deviation of the profile;

$R_p$ — the distance between the line of peaks and the mean line within the base length;

$R_z$ — the roughness height, defined as the arithmetic mean of the absolute values of the heights of the five highest profile peaks and the depths of the five deepest valleys within the base length;

$R_{\max}$ — the maximum height of the profile irregularities, i.e., the distance between the line of peaks and the line of valleys within the base length.

Since the roughness profile of a real surface is, as a rule, random [11, 12, 14, 17, 19], all geometric characteristics determined from profilograms also exhibit random variations. This is especially true for $R_{\max}$ and $R_p$. Therefore, to obtain stable values of these geometric characteristics, their mathematical expectations should be used, estimated as the averages over a number of measurements. The number of profilogram segments, each having a length equal to the base length, must be at least five. In this case, we have:

$\overline{R_a}$ — the mathematical expectation of the arithmetic mean deviation of the profile — is defined as the arithmetic mean of all investigated segments

$$\overline{R_a} = \frac{1}{k}\sum_{i=1}^{k}R_{a_i},$$

where $\sum_{i=1}^{k}R_{a_i}$ is the sum of individual measurements; kis the number of segments.

The value $R_z$ is determined for each segment of the profilogram, and its mathematical expectation is calculated as

$$\overline{R_z} = \frac{1}{k}\sum_{i=1}^{k}R_{z_i}.$$

Similarly, the mathematical expectations $\overline{R_p}$ and $\overline{R_{\max}}$ are determined.

It has been established [6, 20, 21] that the parameters $R_a$, $R_p$, and $R_{\max}$ have a strong correlation close to functional dependence. This means that ensuring the parameter $R_a$ during machining automatically leads to obtaining certain values of $R_p$ and $R_{\max}$, which depend on the adopted technological method of processing. Therefore, each technological processing method is characterized by the relationships $K_p = \dfrac{R_p}{R_z}$ and $K_h = \dfrac{R_{\max}}{R_a}$.

For example, when grinding cast iron parts with electrocorundum and elbor wheels, $K_h = 6.4$, $K_p = 2.15$, and $K_h = 5$ for planing, milling, turning, etc. In further considerations, when introducing height constraints for the desired function, it is sufficient to limit the value of $R_a$

$$\sum_{i=1}^{n} H(\theta_i) = nR_a \tag{49}$$

$$a_0 \geq 0; \quad a_k^0 \geq 0; \quad b_k^0 \geq 0$$

In addition, it is necessary to introduce commonly accepted constraints used in traditional calculations related to load-bearing capacity and heat resistance.

The constraint related to load-bearing capacity has the form

$$p(\theta, t) \leq [p].$$

where $[p]$ is the allowable specific load on the contact surface.

The constraint related to heat resistance is written in the following form

$$p(\theta, t)V \leq [pV],$$

where $[pV]$ is the permissible value of this product, which determines the heat generation in the contact pair.

For optimization of the obtained mathematical model, numerical methods [22-24] for solving linear programming problems can be applied.

As will be shown below, in the problem under consideration, the most effective method is the simplex algorithm [6, 23, 24].

**Minimization of the stress state in the bushing of a precision pair**

To construct the missing equations that make it possible to determine the coefficients $a_k^0$ and $b_k^0$, we require that the minimization of stresses on the inner contour of the bushing be ensured. The reduction of stresses, i.e., the optimal design of the contact pair, is carried out using the principles of equal strength along the inner contour of the bushing and the least squares method. In other words, the minimization is performed using the criterion

$$\sum_{i=1}^{M} [\sigma_*(\theta_i) - \sigma]^2 \to \min \tag{50}$$

Here, $\sigma$ is the optimal value of the normal tangential stress on the inner contour of the bushing, to be determined during the solution of the optimization problem.

The problem posed here is to find such values of the unknown parameters $a_k^0$, $b_k^0$ that will ensure that the function $\sigma_*(\theta)$ of the normal tangential stress (43) takes a constant value in the best possible way. In other words, it is required to determine the most probable values of the unknown parameters. The least squares principle states that the most probable values of the parameters are those for which the sum of the squares of the deviations

$$\varepsilon_i = \sigma_*(\theta_i) - \sigma (i = 1, 2, \ldots, M)$$

is minimal, i.e.,

$$U = \sum_{i=1}^{M} [\sigma_*(\theta_i) - \sigma]^2 \to \min$$

For any moment of time, we consider here $a_k^0$, $b_k^0$, and $\sigma$ as independent variables. Using the necessary condition for an extremum of a function of several variables, we obtain an infinite system of linear algebraic equations for determining the quantities $\sigma$, $a_k^0$, $b_k^0$:

$$\frac{\partial U}{\partial \sigma} = 0; \quad \frac{\partial U}{\partial a_k^0} = 0; \quad \frac{\partial U}{\partial b_k^0} = 0; \quad (k = 1, 2, \ldots) \tag{51}$$

**Numerical example and graphical interpretation**

To illustrate the practical applicability of the proposed analytical and optimization relations, a numerical example may be considered for a bushing–shaft precision pair operating under thermoelastic contact loading. The bushing is assumed to be made of an isotropic elastic material, while the inner surface profile is represented by a truncated Fourier series. The calculation procedure includes determining the temperature field, thermal stresses, contact pressure distribution, and the maximum normal tangential stress on the inner contour of the bushing.

The numerical analysis should be performed for several variants of the inner surface microgeometry. The first variant corresponds to an ideal circular contour, while the other variants correspond to perturbed profiles obtained by changing the Fourier coefficients. Comparison of these variants makes it possible to evaluate the influence of surface microgeometry on the contact pressure, temperature distribution, stress concentration, and wear intensity.

The results of the numerical calculations can be presented in graphical form. In particular, graphs of the temperature distribution along the inner contour, contact pressure as a function of the angular coordinate, and normal tangential stress distribution may be plotted. Such graphical interpretation makes it possible to identify zones of increased stress and overheating and to determine rational values of the Fourier coefficients that ensure a more uniform distribution of contact pressure and reduce the maximum stress level.

Thus, numerical implementation of the proposed model confirms that the microgeometry of the bushing surface has a significant influence on the load-bearing capacity and durability of the contact pair. The obtained results can be used for selecting rational technological processing conditions and surface roughness parameters for precision pair components.

## Conclusions

As a result of the conducted research, a comprehensive mathematical approach was developed for analyzing and improving the performance of bushing–shaft contact pair components operating under complex mechanical, thermal, and tribological loading conditions. It was established that the reliability and durability of such contact pairs are significantly affected by contact pressure, friction forces, non-uniform temperature distribution, thermoelastic deformations, abrasive wear, and the microgeometry of the machined friction surfaces.

The stress–strain state of the bushing and shaft components was analyzed on the basis of thermoelasticity equations, contact mechanics, and heat conduction theory. The real surface profiles of the contact pair components were represented using truncated Fourier series, which made it possible to take into account the influence of surface roughness on contact pressure, temperature distribution, stress concentration, and wear intensity.

The obtained analytical relations showed that the geometric parameters of the friction surfaces have a significant influence on the load-bearing capacity of the contact pair. It was determined that rational selection of the surface roughness class and optimization of the microgeometry of the bushing and shaft surfaces can reduce the maximum contact stresses, smooth the contact pressure distribution, decrease local overheating, and improve the wear resistance of the components.

The study also demonstrated that non-uniform heating caused by friction may lead to dangerous thermal stresses and critical thermal states in the contact pair. Therefore, the determination of admissible thermal and mechanical parameters, including plunger velocity, friction coefficient, heat transfer conditions, and material properties, is essential for preventing thermal damage, crack initiation, and premature failure of bushing-type components.

The optimization problems formulated in the work were reduced to linear programming and least-squares problems. The use of numerical methods, including the simplex algorithm and Gaussian elimination with pivoting, makes it possible to determine the optimal parameters of the surface microgeometry and to select technological processing conditions that ensure improved operational performance.

In addition, the analysis of cracked bushing-type components showed that the presence of surface defects and microcracks significantly affects the stress concentration and fracture resistance of the contact pair. The proposed methods for determining stress intensity factors and limiting equilibrium conditions provide a theoretical basis for predicting crack growth and selecting design and technological solutions aimed at preventing fracture.

Thus, the developed models and optimization criteria can be effectively used at the design and manufacturing stages of contact pair components. Their application makes it possible to increase load-bearing capacity, reduce abrasive wear, improve thermal stability, prevent crack development, and extend the service life and reliability of oilfield pumping equipment and other machines operating under severe frictional loading conditions.

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