



A NUMERICAL METHOD FOR RESTORING PRESSURE DISTRIBUTION IN THE GAS REGIME OF RESERVOIR DEVELOPMENT BASED ON THE MODEL OF TURBULENT FILTRATION

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ABSTRACT

The process of developing a rectangular gas reservoir in the gas regime is considered. A one-dimensional model of is proposed to describe this process. The pressure distribution in the reservoir at the initial moment of time, the pressure and volume flow rate of gas in the production gallery of wells are considered to be set. However, data on the pressure and flow of gas at the outer boundary of the reservoir are assumed to be unknown. Within the framework of the proposed model, the task is to determine the pressure distribution in the reservoir only based on the information specified in the operational gallery. This problem belongs to the class of boundary inverse problems. First, a discrete analogue of the problem is constructed using the method of difference approximation. To numerically solve the resulting system of difference equations, a special computational algorithm is proposed based on the use of representation for solving a system of linear algebraic equations with a tridiagonal matrix. As a result, an explicit formula was obtained for determining the pressure value at the outlet boundary of the reservoir and a recurrent formula for determining the pressure distribution in the reservoir at each time layer. To ensure the stability of the solution of the inverse problem, the method of natural regularization is used. Numerical experiments for a model gas reservoir were carried out based on the proposed computational algorithm.

Keywords: gas regime of reservoir development; rectilinearly parallel gas flow; boundary inverse problem; self-regularization method; difference approximation method.

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Introduction

It is known that in the process of developing gas fields, depending on the form of reservoir energy that causes the movement of gas to production wells, water and gas regimes are mainly encountered. When developing gas deposits in a water-drive regime, the main source of reservoir energy is the pressure of marginal (plantar) waters. In the gas regime of development, the movement of gas to wells occurs due to the potential energy of gas expansion with a decrease in reservoir pressure [1, 2].

It should be noted that reservoir pressure, which determines the energy potential of a gas-bearing reservoir, is one of the main characteristics of the reservoir and affects almost all indicators of reservoir development, including well productivity. Therefore, during the development of a gas-bearing reservoir, constant monitoring of the dynamics of pressure distribution in the reservoir is required. For this purpose, the mathematical model of an unsteady single-phase filtration flow in a reservoir is mainly used in the practice of developing gas fields [3–6]. This model includes the differ-

ential equation of continuity of a single-phase gas filtration flow, the equation of motion of the gas flow, which uses one or another empirical filtration law, and additional equations of state for gas and a porous medium. In practice, when modeling a single-phase filtration flow in a reservoir, as an equation of motion, Darcy's law is used, which expresses the linear dependence of the filtration rate on the pressure gradient. It is known that Darcy's law is valid only in a certain limited range of filtration rates, in which turbulence, inertia, and other high-speed effects are negligible. Numerous experimental studies show that as the filtration rate increases, due to the turbulence of the fluid flow, a deviation from Darcy's law occurs. It is believed that this range of filtration rates can be adequately represented by the so-called law of turbulent filtration, which describes both laminar flow and flow with inertial-turbulent effects in the reservoir [2, 7].

Obviously, in order to solve practical problems related to the development of formations in the gas regime, in addition to the mathematical model, it is necessary to know the initial and boundary conditions describing the initial state of the formation and the interaction of the formation with its surrounding area. However, it should be noted that access to the gas reservoir is limited and is possible only through

production wells that open the reservoir in small areas. Therefore, the pressure and gas flow at the outer boundary of the reservoir are not available for direct measurements. Therefore, it is practically impossible to accurately represent the conditions at the outer boundary of the reservoir. The main sources of information about the processes occurring in the reservoir during development are production wells, where the pressure and volume flow rate of gas are available for direct measurements. In this regard, for the practice of developing gas fields, the task of determining the pressure distribution in the reservoir only on the basis of information obtained from production wells is of great importance.

Statement of the problem

Let us consider a horizontally located gas reservoir in the form of a rectangular parallelepiped with a length L , of constant width d and low thickness h . A reservoir bounded from above and below by impenetrable planes is considered a homogeneous, non-deformable porous medium. A gallery of production wells is located at the entrance boundary of the reservoir $x=0$. At the initial moment of time $t=0$, the well gallery is put into operation and occurs gas regime of reservoir development. It is assumed that in the process of reservoir development, according to the law of turbulent filtration, an isothermal rectilinearly parallel gas flow occurs to the production gallery.

Let's make a mathematical model of the gas regime of the development of this reservoir, which includes: the differential equation of continuity of the filtration flow of gas

$$\phi \frac{\partial \rho(x,t)}{\partial t} = \frac{\partial \rho(x,t)u(x,t)}{\partial x} \tag{1}$$

the equation of motion of the filtration gas flow, which is the law of turbulent filtration

$$\frac{\partial P(x,t)}{\partial x} = \frac{\mu}{k} u(x,t) + \chi \rho(x,t) u^2(x,t) \tag{2}$$

and the equation of state of a perfect gas

$$\rho(x,t) = \frac{\rho_{at}}{p_{at}} P(x,t) \tag{3}$$

where $P(x,t)$ is the reservoir pressure, $u(x,t)$ is the gas flow velocity, $\rho(x,t)$ is the gas density, ϕ is the porosity coefficient, μ is the dynamic viscosity of the gas, k is the absolute permeability of the reservoir, χ is the turbulence coefficient, ρ_{at} is the density of the gas at atmospheric pressure p_{at} .

First, we transform the system of equations (1)–(3). For this purpose, we multiply both parts of equation (2) by the gas density $\rho(x,t)$

$$\rho(x,t) \frac{\partial P(x,t)}{\partial x} = \frac{\mu}{k} \rho(x,t) u(x,t) + \chi \rho^2(x,t) u^2(x,t)$$

and taking into account the relationship between the filtration rate and the gas mass flow rate $G(x,t) = S \rho(x,t) u(x,t)$, we reduce the system of equations (1)–(3) to two equations

$$\frac{\rho_{at} \phi}{p_{at}} \frac{\partial P(x,t)}{\partial t} = \frac{1}{S} \frac{\partial G(x,t)}{\partial x}, \tag{4}$$

$$\frac{\rho_{at}}{p_{at}} P(x,t) \frac{\partial P(x,t)}{\partial x} = \frac{\mu}{kS} G(x,t) + \frac{\chi}{S^2} G^2(x,t), \tag{5}$$

$$0 < x < L, \quad 0 < t \leq T$$

where $S=dh$ is the cross-sectional area of the formation. Now we reduce the system of equations (4) and (5) to a single equation with respect to reservoir pressure $P(x,t)$. Having solved equations (5) with respect to the function $G(x,t)$, we have

$$G(x,t) = \theta + \sqrt{\theta^2 + \lambda \frac{\partial P^2(x,t)}{\partial x}} \tag{6}$$

$$\text{where } \theta = -\frac{S\mu}{2k\chi}, \quad \lambda = \frac{S^2 \rho_{at}}{2\chi p_{at}}.$$

Putting the expression $G(x,t)$ in equation (4), we obtain a nonlinear parabolic equation with respect to the function $P(x,t)$

$$\frac{2\phi\chi}{S} \sqrt{\theta^2 + \lambda \frac{\partial P^2(x,t)}{\partial x}} \frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x} \left(P(x,t) \frac{\partial P(x,t)}{\partial x} \right), \tag{7}$$

$$0 < x < L, \quad 0 < t \leq T$$

Thus, numerical simulation of the gas regime of the reservoir, accompanied by turbulent filtration, will be based on solving equations (6) and (7) under appropriate initial and boundary conditions.

Let the pressure distribution in the reservoir be known at the initial moment of time $t=0$, i.e. for equation (7) we have the following initial condition

$$P(x,0) = \psi(x) \tag{8}$$

Assuming that the volume flow rate of gas in the production gallery is set, we will have a condition for the mass flow rate of gas $G(x,t)$ at the inlet boundary of the reservoir $x=0$.

$$G(0,t) = \rho_{at} Q(t)$$

where $Q(t)$ is the volume flow rate of gas in the production gallery, adjusted to atmospheric conditions. Then, at the input boundary of the reservoir, according to the representation of the law of turbulent filtration (5), we will have the following boundary condition for equation (7)

$$\frac{P(0,t)}{p_{at}} \frac{\partial P(0,t)}{\partial x} = \frac{\mu}{kS} Q(t) + \frac{\chi \rho_{at}}{S^2} Q^2(t) \tag{9}$$

Obviously, in order to correctly formulate the boundary value problem for equation (7), it is necessary to have a boundary condition at the output boundary of the reservoir $x=L$. However, due to the fact that the pressure and filtration flow of gas at the outlet boundary of the reservoir are not available for direct observation, it is not possible to formulate a condition at this boundary. Suppose that instead of the missing information at the outlet boundary of the reservoir, an additional condition is set in the production gallery regarding pressure

$$P(0,t) = W(t) \tag{10}$$

where $W(t)$ is the given function.

Thus, the task of determining the dynamics of the pressure distribution in the reservoir is reduced to solving equation (7) under conditions (8)–(10). It should be noted that the boundary value problem (7)–(10), the solution of which determines the pressure distribution in the reservoir, is an ill-posed problem due to the assignment of additional condition (10) at the input boundary of the reservoir $x=0$ and belongs to the class of boundary inverse problems [8, 9].

Theoretical issues related to the correctness of boundary value inverse problems, the existence and uniqueness of solutions to a class of boundary value inverse problems for equations of mathematical physics in various functional spaces are studied in [10-14]. Many works have been devoted to the development and substantiation of computational algorithms for the numerical solution of boundary value inverse problems. [8, 9, 15–22].

Solution method

To numerically solve the inverse problem (7)–(10), we first construct its discrete analog using the method of difference approximation. To this end, we discretize a given rectangular area $\{0 \leq x \leq L, 0 \leq t \leq T\}$ in space and in time, with steps $\Delta x = \frac{L}{n}$ by variable x and $\Delta t = \frac{T}{m}$ by variable t

$$\bar{\omega} = \{(x_i, t_j) : x_i = i\Delta x, t_j = j\Delta t, i = 0, 1, 2, \dots, n, j = 0, 1, 2, \dots, m\}$$

Using an explicit time approximation for the nonlinear coefficients in equation (7), we represent the discrete model of problem (7)–(10) on a grid $\bar{\omega}$ as

$$\frac{2\phi\chi}{S} \sqrt{\theta^2 + \lambda \frac{(P_i^{j-1})^2 - (P_{i-1}^{j-1})^2}{\Delta x} \frac{P_i^j - P_{i-1}^{j-1}}{\Delta t}} = \frac{1}{\Delta x} \left[P_{i+1/2}^{j-1} \frac{P_{i+1}^j - P_i^j}{\Delta x} - P_{i-1/2}^{j-1} \frac{P_i^j - P_{i-1}^j}{\Delta x} \right], \quad (11)$$

$$i = \overline{1, n-1}, \quad j = \overline{1, m},$$

$$P_i^0 = \psi_i, \quad i = \overline{0, n}, \quad (12)$$

$$\frac{P_0^{j-1}}{p_{at}} \frac{P_1^j - P_0^j}{\Delta x} = \frac{\mu}{kS} Q^j + \frac{\chi p_{at}}{S^2} (Q^j)^2, \quad j = \overline{1, m}, \quad (13)$$

$$P_0^j = W^j, \quad j = \overline{1, m}, \quad (14)$$

where $P_i^{j-1} \approx P(x_i, t_j)$, $P_{i\pm 1/2}^{j-1} = (P_i^{j-1} + P_{i\pm 1}^{j-1}) / 2$, $Q^j = Q(t_j)$,

$$W^j = W(t_j), \quad \psi_i = \psi(x_i).$$

As can be seen, the discrete model (11)–(14), for each fixed value of $j, j=1, 2, \dots, m$, is a system of linear difference equations in which the unknowns are $P_i^j, i=1, 2, \dots, n$, i.e. approximate values of the desired function $P(x, t)$ at the nodal points of the difference grid $\bar{\omega}$.

It is known that one of the widespread methods for finding a stable solution to discrete analogues of inverse problems is Tikhonov’s local regularization method, the main idea of which is to reduce the inverse problem to the problem of minimizing a certain functional with a local regularizing term [8, 9]. Obviously, by introducing a smoothing functional instead of condition (14), the solution to the system of difference equations (11)–(13) for each fixed value of $j, j=1, 2, \dots, m$, can be represented as a solution to a variational problem with local regularization. However, when applying the regularization method, a non-trivial problem of choosing the optimal regularization parameter arises. The value of this parameter must be consistent with the error in the input data. In this regard, to find a stable solution to the system of difference equations (11)–(14), we use the self-regularization (natural regularization) method [8, 9]. The advantage of this method is that the inverse problem is solved numerically in its original

formulation, using the viscosity properties of computational algorithms for solving the system of difference equations (11)–(14). We will use the time discretization step Δt as the regularization parameter.

We write the systems of difference equations (11)–(14) in the following form

$$a_i P_{i-1}^j - c_i P_i^j + b_i P_{i+1}^j = -F_i^{j-1}, \quad (15)$$

$$i = 1, 2, \dots, n-1,$$

$$P_i^0 = \psi_i, \quad i = \overline{0, n}, \quad (16)$$

$$P_0^j = P_1^j - \left[\frac{\mu}{kS} Q^j + \frac{\chi p_{at}}{S^2} (Q^j)^2 \right] \frac{p_{at} \Delta x}{P_0^{j-1}}, \quad (17)$$

$$P_0^j = W^j, \quad j = 1, 2, \dots, m, \quad (18)$$

where $a_i = \frac{P_{i-1/2}^{j-1}}{\Delta x^2}$, $b_i = \frac{P_{i+1/2}^{j-1}}{\Delta x^2}$, $c_i = a_i + b_i + R_i^{j-1}$, $F_i^{j-1} = R_i^{j-1} P_i^{j-1}$,

$$R_i^{j-1} = \frac{2\phi\chi \sqrt{\theta^2 + \lambda [(P_i^{j-1})^2 - (P_{i-1}^{j-1})^2] / \Delta x}}{S\Delta t}$$

It is obvious that the system of linear difference equations (15)–(18) for each fixed value of $j, j=1, 2, \dots, m$, has a tridiagonal matrix. Therefore, the solution of the system of equations (15)–(18) for each time layer j can be represented as

$$P_i^j = \alpha_{i+1} P_{i+1}^j + \beta_{i+1}, \quad i = 0, 1, 2, \dots, n-1 \quad (19)$$

where $\alpha_{i+1}, \beta_{i+1}$ are unknown coefficients. Let’s write a similar expression for P_{i-1}^j :

$$P_{i-1}^j = \alpha_i P_i^j + \beta_i = \alpha_i \alpha_{i+1} P_{i+1}^j + \alpha_i \beta_{i+1} + \beta_i$$

Substituting the expressions P_i^j and P_{i-1}^j into equation (15), we obtain the following nonlinear equations for determining the coefficients $\alpha_{i+1}, \beta_{i+1}$

$$\alpha_{i+1} = \frac{b_i}{c_i - a_i \alpha_i}, \quad \beta_{i+1} = \frac{a_i \beta_i + F_i^{j-1}}{c_i - a_i \alpha_i}, \quad i = 1, 2, \dots, n-1$$

We find the initial values of the coefficients α_i, β_i from the requirement of equivalence of relation (19) at $i=0$, i.e. $P_0^j = \alpha_1 P_1^j + \beta_1$, to condition (17)

$$\alpha_1 = 1, \quad \beta_1 = - \left[\frac{\mu}{kS} Q^j + \frac{\beta p_{at}}{S^2} (Q^j)^2 \right] \frac{p_{at} \Delta x}{P_0^{j-1}}$$

Once the coefficients α_i, β_i have been found for all $i = 1, n$, we can define the dependence P_0^j on P_n^j in explicit form. To do this, we write equation (19) for $i=0$

$$P_0^j = \alpha_1 P_1^j + \beta_1$$

Substituting here the expression for P_1^j , i.e. $P_1^j = \alpha_2 P_2^j + \beta_2$, we have

$$P_0^j = \alpha_1 \alpha_2 P_2^j + \alpha_1 \beta_2 + \beta_1$$

Further, substituting expressions $P_2^j, P_3^j, \dots, P_{n-1}^j$ into the last equation, we obtain the desired relationship between P_0^j and P_n^j explicitly

$$P_0^j = P_n^j \prod_{i=1}^n \alpha_i + \sum_{i=2}^n \beta_i \prod_{l=1}^{i-1} \alpha_l + \beta_1$$

From this we obtain a formula for determining the approximate value of the desired pressure at the outlet boundary of the reservoir on a temporary layer j

$$P_n^j = \frac{W^j - \sum_{i=2}^n \beta_i \prod_{l=1}^{i-1} \alpha_l - \beta_1}{\prod_{i=1}^n \alpha_i} \quad (20)$$

Having determined P_n^j by the formula (20), then by the recurrent formula (19), starting from $i=n-1$, we can calculate sequentially $P_{n-1}^j, P_{n-2}^j, \dots, P_0^j$.

Once the pressure distribution in the reservoir on the time layer j is obtained, the distribution of the gas mass flow rate on the same time layer can be found

$$G_i^j = \theta + \sqrt{\theta^2 + \lambda \frac{(P_i^{j-1})^2 - (P_{i-1}^{j-1})^2}{\Delta x}}, \quad i = 1, 2, \dots, n$$

Thus, the proposed numerical method makes it possible to determine the distribution of pressure and mass flow of gas in the reservoir in each time layer.

The results of numerical calculations

The proposed computational algorithm was tested for data of a model gas reservoir with the following characteristics:

- the length of the reservoir $L=500$ m;
- the width of the reservoir $d=100$ m;
- layer thickness $h=10$ m;
- the porosity coefficient of the formation $\phi=0.25$;
- the absolute permeability coefficient of the formation $k=20 \cdot 10^{-12}$ m²;
- initial reservoir pressure $\Psi(x)=180$ atm.;
- reservoir gas density in atmospheric conditions ($p_{at}=10^5$ Pa) $\rho_{at}=0.8$ kg/m³;

- volume flow rate of gas in atmospheric conditions $Q(t)=120000$ m³/day;
- dynamic viscosity of reservoir gas $\mu=0.12 \cdot 10^{-4}$ Pa·s;
- turbulence coefficient $\chi=10^4$ m⁻¹.

Numerical experiments were carried out according to the following scheme:

I. The pressure at the outlet boundary of the reservoir is set $P(L, t)=P_L(t)$ and the solution of equation (7) satisfying this condition and conditions (8), (9) is determined, i.e. the direct problem for determining the function $P(x, t), 0 < x \leq L, 0 < t \leq L$, is solved.

II. In the course of solving the direct problem, a dependence $W(t)=P(0, t)$ is determined and this dependence is used as an additional condition (10) to solve the inverse problem (7)–(10) to restore pressure, first at the outlet boundary of the reservoir, and then over the entire reservoir area.

Numerical experiments were carried out on a uniform space-time difference grid with steps of $\Delta x=5$ m, $\Delta t=4; 6; 10$ s. $P_L(t)=180+10 \sin 5t$ atm was adopted as a function $P_L(t)$. The results of numerical experiments on restoring the time dynamics of pressure at the outlet boundary of the reservoir are presented in table 1 (columns 2–5). In it, t_j is the time, $P_L(t_j)$ and P_n^j are the exact and calculated values of pressure at the time t_j at the outlet boundary of the reservoir.

When conducting numerical experiments, it was found that at small time discretization steps ($\Delta t=4$ s) an unstable solution appears and the pressure at the outlet boundary of the reservoir is determined by a very large error. However, with an increase in the time discretization step ($\Delta t \geq 4$ s), the accuracy of pressure recovery at the boundary increases. The results of numerical calculations show that when using

Results of numerical experiments on the reconstruction of pressure dynamics at the reservoir outlet boundary

Table 1

$t_j, \text{ min}$	$P_L(t_j), \text{ atm}$	$P_n^j, \text{ atm}$			$P_L(t_j), \text{ atm}$	$P_n^j, \text{ atm}$		
		$\Delta t=4$	$\Delta t=6$	$\Delta t=10$		$\Delta t=4$	$\Delta t=6$	$\Delta t=10$
1	170.00	172.05	169.93	170.00	180.00	181.24	179.93	180.00
2	180.44	181.11	180.37	180.44	180.00	181.10	179.99	180.00
3	189.98	187.83	189.99	189.98	180.00	181.11	180.02	180.00
4	179.12	177.63	179.10	179.12	180.00	178.75	180.02	180.00
5	170.06	168.66	170.04	170.06	180.00	181.06	180.11	180.00
6	181.32	181.05	181.36	181.32	180.00	178.68	179.98	180.00
7	189.88	192.09	189.85	189.88	179.99	179.53	179.96	179.99
8	178.24	178.47	178.27	178.24	179.99	182.61	179.91	179.99
9	170.20	166.87	170.21	170.20	179.99	180.80	180.00	179.99
10	182.19	181.66	182.12	182.19	179.99	183.55	180.05	179.99
11	189.71	188.20	189.68	189.71	179.99	181.61	180.04	179.99
12	177.38	178.39	177.35	177.38	179.99	181.37	180.00	179.99
13	170.41	171.93	170.39	170.41	179.99	176.97	179.93	179.99
14	183.04	185.78	183.12	183.04	179.98	182.90	179.98	179.98
15	189.46	187.52	189.39	189.46	179.98	177.31	179.88	179.98
16	176.54	177.60	176.57	176.54	179.98	180.71	180.04	179.98
17	170.70	170.53	170.73	170.70	179.98	178.14	180.02	179.98
18	183.87	184.88	183.84	183.87	179.98	179.33	179.91	179.98
19	189.13	188.25	189.18	189.13	179.98	177.06	179.92	179.98
20	175.72	176.76	175.67	175.72	179.98	180.38	179.88	179.98

$\Delta t=4$ s and $\Delta t=6$ s, the maximum relative error of pressure recovery at the boundary does not exceed 1.95 and 0.044 %, respectively. At $\Delta t \geq 10$ s the pressure at the boundary is recovered exactly.

According to the proposed scheme, a numerical experiment was also conducted for a gas reservoir with an impenetrable outlet boundary. In this case, during the solution of the direct problem, the condition of non-leakage at the outlet boundary of the reservoir $\partial P(L,t)/\partial x=0$ is used. The results of the numerical calculations performed for this reservoir are also presented in table 1 (columns 6-9). It follows from the table that when using $\Delta t=4$ s and $\Delta t=6$ s, the maximum relative error in restoring pressure at the outlet boundary of the reservoir does not exceed 1.98 and 0.064 %, respectively. And with $\Delta t=10$ s the pressure at the border is restored exactly.

Table 2 shows the results of numerical experiments to restore the pressure distribution in a reservoir with an impenetrable outlet boundary, at time points 1 day, 10 day, 30 days, 60 days, calculated at $\Delta t=10$ s. In it \bar{P}, \tilde{P} are the exact and calculated values of the function $P(x,t)$ at the point x at the moment of time t .

The obtained results show that the dynamics of pressure distribution in the reservoir is restored exactly. Therefore, at each time point, the exact and calculated pressure distributions in the reservoir are presented in the same column of table 2.

An analysis of the results of the numerical experiments shows that the proposed numerical method can be used to control the dynamics of pressure distribution during the development of gas fields.

Results of numerical experiments on reconstructing the dynamics of pressure distribution in the reservoir				
x, m	\bar{P}, \tilde{P}, atm			
	$t=1$	$t=10$	$t=30$	$t=60$
0	178.053	160.597	121.805	63.612
25	178.054	160.598	121.806	63.615
50	178.055	160.599	121.808	63.618
75	178.056	160.600	121.809	63.621
100	178.057	160.601	121.811	63.623
125	178.058	160.602	121.812	63.626
150	178.059	160.603	121.813	63.628
175	178.060	160.604	121.814	63.630
200	178.060	160.605	121.815	63.632
225	178.061	160.606	121.816	63.634
250	178.062	160.606	121.817	63.636
275	178.062	160.607	121.818	63.637
300	178.063	160.607	121.819	63.639
325	178.063	160.608	121.820	63.640
350	178.063	160.608	121.821	63.641
375	178.064	160.609	121.821	63.642
400	178.064	160.609	121.821	63.643
425	178.064	160.609	121.821	63.643
450	178.064	160.609	121.821	63.643
475	178.064	160.609	121.821	63.644
500	178.064	160.609	121.821	63.644

Conclusions

The problem of determining the pressure distribution during the development of a rectangular gas reservoir in the gas regime, according to a given volume flow rate of gas and pressure in the production well gallery, is considered. To solve this problem:

1. Based on the initial mathematical model of the gas regime of development of a rectangular formation (1)–(3), a model was constructed that describes the dynamics of the distribution of formation pressure in the form of a nonlinear parabolic equation, as well as the mass flow rate of gas in the formation;
2. For the constructed model, a boundary inverse problem is formulated and a special computational algorithm is developed to solve the problem.

The constructed mathematical model and the proposed computational algorithm make it possible to obtain a dynamic picture of the change in reservoir pressure during the development of a gas reservoir, based only on the available information in the gallery of production wells.

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