

TECHNOLOGICAL EFFICIENCY AND PRODUCTIVITY ANALYSIS IN SUCKER-ROD PUMPING WELLS

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ABSTRACT

Rod pumping systems represent a fundamental component of artificial lift technologies, particularly in the development and exploitation of mature oil fields. Despite their widespread use, the quantitative evaluation of their operational efficiency remains a challenging task, largely due to the nonlinear dynamics of well behavior and the inherent limitations of conventional diagnostic methods. This study presents a comprehensive and structured methodology for evaluating the efficiency of sucker-rod pumping wells by integrating field measurements with advanced computer-based simulation techniques. The proposed framework is based on directly measurable operational parameters in order to formulate a practically applicable concept for evaluating the efficiency of pump operation. Within this framework, efficiency is conceptualized as a function of reservoir productivity, pump fillage factor, production rate, stroke speed, the geometric characteristics of the pump, and other indicators that collectively characterize the well–reservoir system. The proposed methodology is demonstrated under various geo-technological conditions using a computer simulator of the pump–well–reservoir system developed on the basis of the authors' Discrete-Imitation Modeling Concept. It provides a robust analytical basis for evaluating the efficiency of production using sucker-rod pumps and supports the development of informed strategies for production enhancement. The notions of the Pump Productivity Index and the Technological Efficiency Coefficient are introduced, and analytical expressions for efficiency evaluation are presented. Using the simulator, a methodology for the investigation and optimization of the sucker-rod pump well–reservoir system is demonstrated. Validation using data from a real field well confirms the methodological soundness and practical relevance of the approach, demonstrating its potential as an effective decision-support tool for petroleum engineers engaged in the optimization of sucker-rod pump systems.

Keywords: sucker-rod pump; efficiency conception; technological efficiency; pump performance indicator; production data analysis; optimization.

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1. Introduction

Sucker-rod pumping (SRP) systems remain one of the most widely applied artificial lift methods in the global petroleum industry, particularly in mature oil fields with declining reservoir pressure. Their robustness, relative simplicity, and adaptability to a wide range of well conditions have made SRPs the dominant technology for sustaining production in low- and medium-rate wells [1-4]. Despite their extensive use, however, the objective quantitative assessment of operational efficiency in sucker-rod-pumped wells remains problematic at the conceptual level.

Traditional diagnostic tools, such as the interpretation of dynamometer cards, provide valuable insights into pump behavior. Nevertheless, these methods are largely limited to qualitative assessments and represent only short-term snap-

shots of well operation [5]. Likewise, efficiency indicators derived from volumetric calculations are useful in practice but fail to capture the system as a whole. In particular, they neglect reservoir–well interactions and transient processes. Most importantly, these approaches do not explicitly reflect the relationship between production performance and energy consumption. It is important to emphasize a key nuance here: within these approaches, the discussion often concerns not the operational efficiency of the system, but rather the productivity of the pump. These concepts, however, are fundamentally different. The critical issue lies in how productivity is achieved, that is, at what level of efficiency. A sound operating strategy should aim to achieve maximum productivity while ensuring high efficiency. In this context, the need arises to develop a unified concept that links productivity and efficiency in sucker-rod pump operations, as well as a corresponding methodology for efficiency evaluation within this conceptual framework.

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The development of digital technologies has led to the emergence of computer-based modeling and simulation tools, including integrated pump–well–reservoir system simulators [6-9]. These tools provide extensive capabilities for detailed analysis and systematic investigation of sucker-rod pumping operations. However, in the absence of a clearly defined concept of efficiency as a physical and operational parameter, their application for selecting optimal operating regimes remains limited and often relies on subjective judgment.

Historically, the evaluation of sucker-rod pumping systems has been approached primarily from a diagnostic perspective. This has shaped the current state of research, where the vast majority of studies focus on pump diagnostics rather than efficiency assessment (e.g., [10-13]). Consequently, efficiency in sucker-rod pumping practice is commonly defined as the ratio of actual production to theoretical pump capacity. In essence, this definition characterizes pump performance rather than true operational efficiency. A high pump production rate does not necessarily imply high efficiency. In some cases, a deliberate reduction in production rate can lead to a significant improvement in overall efficiency. Therefore, within the considered context, efficiency must be defined as a function of measurable system indicators rather than as a single performance ratio.

Recent advances in digital oilfield technologies have significantly expanded the role of physics-based simulation, digital twins, and computational decision-support systems in production optimization [14, 15]. Integrated dynamic models are increasingly employed to support sensitivity analysis, operating-regime evaluation, and predictive performance assessment of artificial-lift systems. At the same time, data-driven and machine-learning techniques have opened new possibilities for production forecasting and optimization [16].

Nevertheless, physics-based models remain indispensable for understanding the causal relationships governing reservoir–well–pump interactions and for providing physically interpretable engineering insights [16, 17]. Consequently, hybrid methodologies combining rigorous physical modeling with computational optimization have attracted growing attention in recent years [18]. Despite these advances, there remains a lack of physically interpretable indicators capable of simultaneously characterizing productivity and technological efficiency in sucker-rod pumping systems. Conventional performance measures typically evaluate only individual aspects of system operation and therefore provide limited support for multi-criteria operating-regime selection.

The present study addresses this gap by proposing an integrated productivity–efficiency analysis framework based on two physically interpretable indicators, the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC). Combined with simulation-based analysis, these indicators provide a practical basis for multi-criteria operating-regime selection and optimization of sucker-rod pumping wells.

Building upon this concept, the objective of this study is to develop and demonstrate a general methodology for evaluating technological efficiency and productivity in sucker-rod pumping wells based on the integrated PPI–TEC framework. The concept of efficiency considered herein is limited to technological factors. Once an optimal operating regime is

identified within this framework, economic evaluation can always be performed as a subsequent step. Consequently, the proposed methodology has not only theoretical significance but also clear practical relevance. It is based on directly measurable well parameters and integrated with simulation capabilities, enabling predictive efficiency assessment and informed operating-regime selection. The methodology is demonstrated using data from a real field well.

2. Existing approaches and their conceptual limitations

In sucker-rod pumping wells, the evaluation of production performance and operational efficiency, as well as the selection of an appropriate operating strategy based on these criteria, represents a key objective. The primary goal of such a strategy should be to achieve maximum production with high efficiency. Therefore, establishing a clear relationship between production performance and efficiency remains one of the central challenges in sucker-rod pump operation.

In current field practice, efficiency is commonly defined as the ratio of the actual pump production rate to its theoretical production capacity. For this reason, existing approaches to assessing production efficiency are largely centered on pump performance. Accordingly, the literature widely employs the concepts of Volumetric Efficiency, Mechanical Efficiency, and Energy Efficiency [2]. The first term characterizes the pump's production capacity, the second accounts for mechanical losses within the pumping system, and the third reflects the overall energy balance. In principle, the overall efficiency is represented as a combination of these three components. Although this framework is theoretically sound, its practical implementation in sucker-rod pumping operations based on directly measurable field parameters is not feasible. As a result, practical applications are typically limited to the use of Volumetric Efficiency alone. However, as noted above, volumetric efficiency essentially reflects pump performance—or, in other words, pump effectiveness—rather than the efficiency of the production process as a whole.

The widespread use of the Volumetric Efficiency concept, or equivalently pump effectiveness [12, 19], has led to another important consequence. Specifically, efficiency-related issues (which in practice correspond to pump performance) have traditionally been addressed through diagnostic approaches. In this context, the most commonly applied method is the analysis of dynamometer cards. This technique has long been regarded as a standard diagnostic tool in field practice [5, 20]. Dynamometer card analysis enables direct visualization of pump operating conditions, evaluation of pump fillage, and identification of downhole operating states [4]. The main advantages of this method are its simplicity and interpretability. However, dynamometer analysis reflects only short-term operating conditions and requires accurate surface and downhole measurements for reliable application, which leads to additional operational costs. Although recent advances in computer-based interpretation have partially mitigated these limitations, this method, in principle, does not allow for the evaluation of efficiency in the broader technological sense discussed above.

A substantial body of research has been devoted to the mathematical modeling of sucker-rod pump kinematics. These studies analyze, among other aspects, plunger motion

in cylinders filled with non-Newtonian fluids [21, 22], load and stress distribution along the rod string, and energy transmission from the surface to the plunger [3, 10]. While such models significantly enhance the understanding of pump mechanics and system behavior, they do not provide a direct means for evaluating the efficiency of the production process.

Sucker-rod pumping simulators [6, 7] and integrated production system models [8, 23, 24], which jointly account for reservoir inflow, wellbore hydraulics, and pump mechanics, offer broader opportunities for performance analysis. These tools enable scenario analysis and optimization across a wide range of operating conditions. Nevertheless, the absence of a general concept for evaluating operational efficiency based on measurable field parameters significantly limits their practical applicability.

In recent years, data-driven approaches have attracted increasing attention. These methods employ machine learning techniques and advanced signal processing to analyze surface measurements, acoustic signals, and historical production data [26, 26]. They provide strong predictive and adaptive capabilities that are essential for real-time monitoring and optimization of sucker-rod pumping systems.

An analysis of existing scientific studies indicates the need to develop a unified concept for the objective evaluation of technological efficiency of the production process in sucker-rod pumping wells. This concept should link pump performance with operational efficiency and provide an analytical basis for simulator-based optimization, using the criterion of achieving maximum production with minimum energy consumption.

3. Conceptual framework for technological efficiency evaluation

Previous works and theoretical basis

The methodology proposed in this study builds upon a series of previous works by the authors, which established the theoretical and computational foundation for analyzing and optimizing the sucker-rod pump–well–reservoir system using the authors' Discrete-Imitational Modeling (DIM) concept.

In [7], published in *Petroleum Research*, the DIM concept was introduced for the first time as a theoretical framework for analyzing the coupled dynamics of the sucker-rod pump–well–reservoir system. The proposed concept is based on a time-discrete representation of interacting mechanical and hydrodynamic processes, allowing synchronous modeling of pump plunger motion, wellbore flow behavior, and reservoir inflow. This approach enables a more accurate description of the inherently nonlinear and transient behavior of rod-pumping systems.

To practically implement the DIM concept and verify the validity of its theoretical assumptions, a dedicated computer simulator, X-Oil Laboratory v.5.0.0, was developed. The software provides a full computational realization of the discrete-imitational modeling approach and serves as an applied platform for testing and validating the proposed theory¹. The conducted studies demonstrated that the DIM approach offers a more realistic representation of sucker-rod pump system behavior compared with classical analytical and numerical methods. In particular, the X-Oil Laboratory

v.5.0.0 simulator captures transient interactions between reservoir inflow, pump plunger motion, and wellbore hydrodynamics within a unified computational framework. This capability establishes a robust theoretical and practical basis for performance forecasting, efficiency assessment, and operational optimization of sucker-rod pumping systems.

In a subsequent study presented at the SPE Middle East Artificial Lift Conference [27] and [28], the DIM approach was extended to account for intermittent pumping regimes, showing how the method can be used to optimize pumping schedules and improve production efficiency under variable inflow conditions. This research highlighted the sensitivity of system performance to operational parameters and demonstrated the potential of DIM for scenario analysis and decision support.

Further advancements were made in [6], where the DIM framework was applied to real field conditions in the Bibi-Eibat oil field. That study focused on energy and efficiency optimization, providing practical validation of the model's predictive capabilities and demonstrating its usefulness in identifying inefficiencies and guiding operational adjustments in actual well environments.

Together, these studies² provide the theoretical and computational basis for the present work. They show that discrete-imitational modeling is not only a powerful analytical tool for simulating sucker-rod pump operations but also a practical foundation for developing a quantitative efficiency evaluation methodology. Building upon these results, the present study introduces an integral approach to defining, quantifying, and analyzing pumping efficiency using measurable field parameters and simulation-based forecasting.

Simulation studies and key findings³

To investigate the process under consideration and to analyze the relationships among its key indicators, a dedicated simulation environment was developed. This simulation framework enables the reproduction of the behavior of sucker-rod pumping wells over a wide range of geotechnical parameters and facilitates the study of interactions between pump performance and the principal parameters of the well–reservoir system. To achieve this, the simulator explicitly accounts for the processes occurring in the main system components—including the reservoir drainage zone, the wellbore, and the pump—while capturing the dynamic interactions among these components. As a result, the simulation environment provides a high-fidelity representation of the actual production process.

Primary attention is first given to examining the relationship between the stroke rate, considered as a parameter characterizing energy consumption; the production volume, representing well productivity; and the pump fillage factor, which reflects pump effectiveness. For this purpose, simulation-based experiments were conducted at various pump submergence depths under reservoir conditions of pressure $p=100$ atm, static fluid column height $h_{st}=1199.3$ m, and permeability $k=10$ mD.

² The theoretical principles of the discrete-imitational modeling concept, as well as relevant methodological developments, are presented in the authors' other works (Jamalbayov and Valiyev 2024a, 2024b; Jamalbayov et al. 2024; Jamalbayov and Jamalbayli 2024; Aliev et al 2019; Jamalbayov et al. 2021), which provide the foundational context for the present study

³ All computational experiments conducted within the scope of this study were performed using X-Oil Laboratory v.5.0.0 software (© Mahammad Jamalbayov, 2025).

¹ The simulator was officially registered by the Intellectual Property Agency of the Republic of Azerbaijan on 29 August 2025 under registration number 01/C-14806-25, with copyright held by (c) Jamalbayov M. A.

The results of these simulations are presented in the left panel of table 2 and in figure 1, illustrating the dependence of the daily production volume and pump fillage on the stroke speed. Here, the black curves correspond to a pump submergence depth of 20% of the static liquid column height below the static liquid level in the well; the red curves to 30%; the green curves to 50%; and the blue curves to 70%. Analyzing the curves in figure 1, it becomes evident that the least favorable scenario occurs when the pump submergence depth is 20% (black curves). This configuration yields the smallest maximum daily pumping volume relative to stroke speed compared to the other scenarios. This outcome is expected because, in this scenario, the depression is the smallest due to the highest liquid column height compared to other scenarios. Importantly, this scenario is also characterized by the lowest level of effectiveness. For instance, at the maximum daily pumping volume (approximately 0.9 m³ corresponding to a stroke speed of 8 stroke/min), the pump fillage is only 16%. In contrast, for the cases corresponding to pump submergence depths of 30, 50, and 70 % (red, green, and blue curves), these values were 20, 33, and 42 %, respectively, while the corresponding daily production volumes were 1.27, 1.96, and 2.54 m³, respectively.

A noteworthy observation is that, at any pump submergence depth, the maximum values of daily production and pump fillage do not coincide at the same stroke rate. This is attributed to the low productivity of the reservoir layer. Specifically, increasing the daily production of the well requires an increase in stroke rate up to a certain limit, whereas further increases in stroke rate beyond this limit prevent the plunger of the pump cylinder from being completely filled during a full stroke cycle due to the weak inflow from the layer. To confirm this, analogous simulations were conducted for high-productivity layers with a reservoir pressure of 60 atm and a well drainage zone permeability of 100 mD. Under these conditions, the dependencies of daily production and pump fillage on stroke speed are presented in figure 2.

If, in a low-productivity layer with a high submergence depth (fig. 1, blue curves), the maxima of daily production volume and pump fillage do not coincide with respect to stroke rate, in a high-productivity layer with elevated pressure and permeability, these maxima coincide at nearly the

same stroke rate for all cases except for the minimum pump submergence depth (fig. 2, black curves, submergence depth 10%). Specifically, when the pump submergence depth is 10%, the maximum daily production volume of 3.5 is achieved at approximately 8 strokes per minute, while the corresponding pump fillage is 50%. In this case, the maximum fillage, slightly above 90%, occurs at a stroke speed of 4 strokes per minute, at which the well's daily production volume is approximately 3 m³.

When the pump submergence depth is 15 and 20 % (fig. 2, green and blue curves, respectively), the daily production volume and pump fillage correspond to approximately the same stroke rates, which are 6 and 8 strokes per minute, respectively.

It should be noted that when the pump is submerged to a depth corresponding to 40% of the static fluid column (indicated by the red lines in fig.2), the daily production volume increases with stroke rate, while the pump fillage remains at 100% over practically the entire range of stroke rates. This behavior reflects the high productivity of the layer, as the high inflow rate from the reservoir to the well allows the plunger to completely fill the cylinder regardless of the plunger stroke frequency.

As observed, the pump's performance is directly dependent on both its productivity and fillage. Operation of the pump at high performance is achieved at the stroke speed that maximizes both parameters simultaneously. Therefore, the product of these two parameters can serve as a measure of Pump Performance. Denoting the volume of fluid produced over a day (i.e., 86400 s) as V (m³) and the pump cylinder fillage as F (%), Pump Performance Indicator (PPI) can be expressed by the following formula:

$$PPI = \frac{VF}{100} \tag{1}$$

Since the parameter calculated from Eq. (1) is dimensional, it does not have a universal character and is therefore not suitable for direct comparison of wells operating under different geo-technological conditions. Nevertheless, Eq. (1) may have practical significance in characterizing the behavior of a specific well-pump system within a given layer and production regime. With this consideration, it has been extensively studied under various geo-technological conditions.

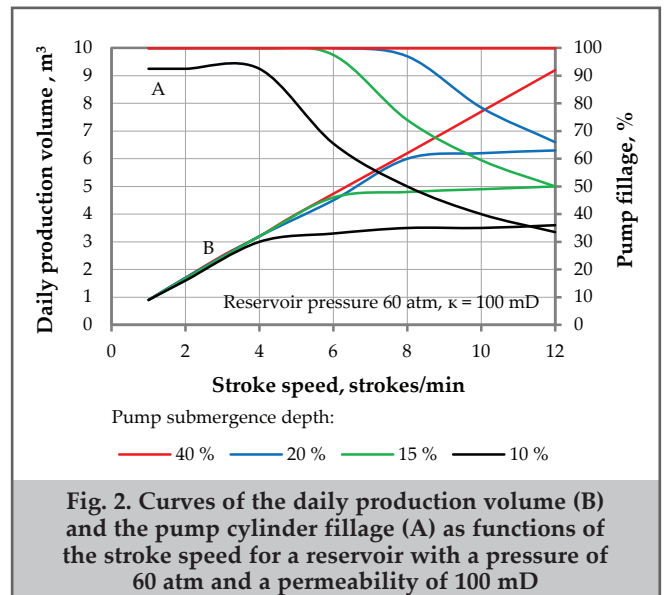
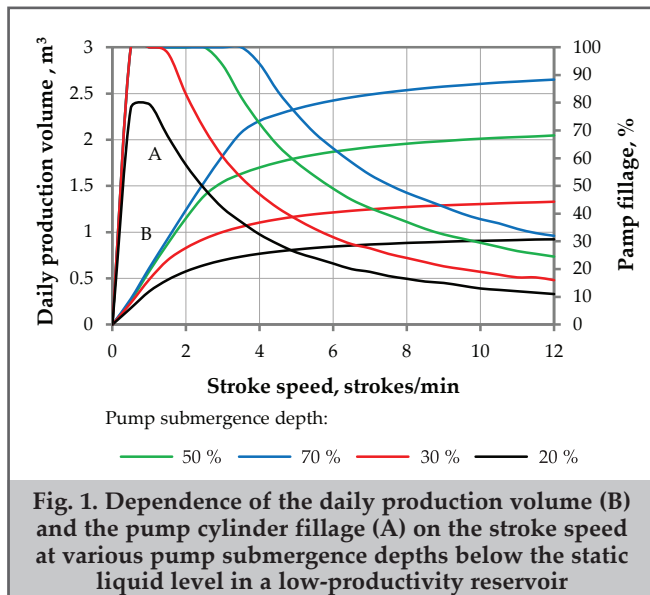


Fig. 1. Dependence of the daily production volume (B) and the pump cylinder fillage (A) on the stroke speed at various pump submergence depths below the static liquid level in a low-productivity reservoir

Fig. 2. Curves of the daily production volume (B) and the pump cylinder fillage (A) as functions of the stroke speed for a reservoir with a pressure of 60 atm and a permeability of 100 mD

The dependence of the parameter calculated from Eq. (1) on stroke speed, for different reservoir pressures, well drainage zone permeabilities, and pump submergence depths, is presented in tables 1 and 2. Table 1 shows the results for a reservoir with a pressure of 60 atm (static fluid column height $h_{st}=719.6$ m) and a permeability of 100 mD, whereas table 2 presents the results for a reservoir with a pressure of 100 atm and permeabilities of 10 mD (left panel) and 100 mD (right panel). In both reservoirs, the production process was simulated for various pump submergence depths. The daily production volume, pump fillage, and the parameter PPI, calculated from Eq. (1), are presented for stroke rates in the range of 1–12 strokes per minute. In both tables, the maximum values of the PPI for each case are highlighted in yellow.

It is noteworthy that, for the reservoir with a pressure of 60 atm, when the pump submergence depth is at its maximum (corresponding to 40% of the static fluid column below the static level), the PPI increases across the entire range of stroke rates (table 1, case 1). In the other cases, PPI begins to decrease after reaching a certain maximum. This behavior can be more clearly observed in figure 3, which shows the PPI versus stroke speed curves (red line). The increase in PPI with stroke speed is due to the high productivity of the reservoir, as evidenced by the data in table 1 (case 1). In this case («Pump submergence depth is 40%»), the pump fillage remains at 100% for any stroke rate, and simultaneously, the daily production volume increases with stroke speed.

Interestingly, a similar trend is not observed in the higher-pressure reservoir (the right panel of table 2, «Pump

Dependence of the produced fluid volume over 86400 s, the pump cylinder fillage factor, and the Pump Performance Indicator (PPI) on the stroke speed for various pump submergence depths, at a reservoir pressure of 60 atm and a permeability of 100 mD				Dependence of the produced fluid volume over 86400 s, the pump cylinder fillage factor, and the Pump Performance Indicator (PPI) on the stroke speed for various pump submergence depths, at a reservoir pressure of 60 atm and a permeability of 100 mD			
P=60 atm, $h_{st}=719.6$ m, $k=100$ mD				P=60 atm, $h_{st}=719.6$ m, $k=100$ mD			
Stroke speed, strokes/min	Daily production volume, m ³	Pump fillage, %	PPI	Stroke speed, strokes/min	Daily production volume, m ³	Pump fillage, %	PPI
Pump submergence depth is 40%				Pump submergence depth is 15%			
1	0.9	100	0.9	1	0.9	100	0.9
2	1.7	100	1.7	4	3.2	100	3.2
3	2.5	100	2.5	6	4.6	97.5	4.485
4	3.2	100	3.2	8	4.8	74	3.552
5	4	100	4	10	4.9	59.5	2.9155
8	6.2	100	6.2	12	5	50	2.5
10	7.7	100	7.7				
12	9.2	100	9.2				
Pump submergence depth is 20%				Pump submergence depth is 10%			
1	0.9	100	0.9	1	0.9	92.5	0.8325
2	1.7	100	1.7	2	1.6	92.5	1.48
4	3.2	100	3.2	4	3	92.5	2.775
6	4.5	100	4.5	6	3.3	65.5	2.1615
8	6	97	5.82	8	3.5	50	1.75
10	6.2	78.5	4.867	10	3.5	40	1.4
12	6.3	66	4.158	12	3.6	33.5	1.206

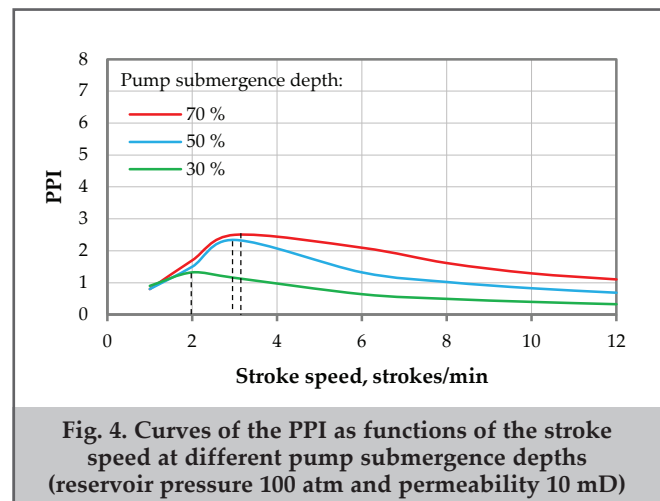
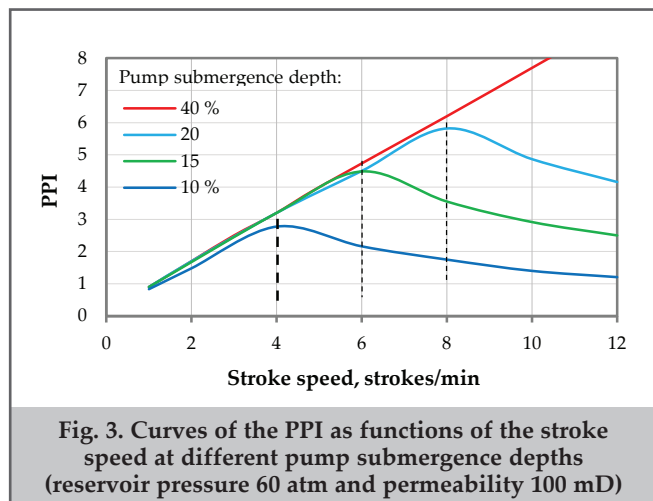


Table 2

Dependence of the produced fluid volume over 86400 s, the pump cylinder fillage factor, and the Pump Performance Indicator (PPI) on stroke speed for various pump submergence depths, at a reservoir pressure of 100 atm and permeabilities of 10 mD (left panel) and 100 mD (right panel)

<i>P</i> =100 atm, <i>h_{st}</i> =1199.3 m, <i>k</i> =10 mD				<i>P</i> =100 atm, <i>h_{st}</i> =1199.3 m, <i>k</i> =100 mD			
Stroke speed, strokes/min	Daily production volume, m ³	Pump fillage, %	PPI	Stroke speed, strokes/min	Daily production volume, m ³	Pump fillage, %	PPI
Pump submergence depth is 70%				Pump submergence depth is 70%			
1	0.61	100	0.61	1	0.8	100	0.80
2	1.24	100	1.24	2	1.7	100	1.70
3	1.83	100	1.83	3	2.5	100	2.50
6	2.42	63.5	1.54	6	3.3	63.5	2.10
8	2.54	47.5	1.20	8	3.4	47.5	1.62
10	2.60	38	0.99	10	3.4	38	1.29
12	2.65	32	0.85	12	3.5	31.5	1.10
Pump submergence depth is 50%				Pump submergence depth is 50%			
1	0.58	100	0.58	1	0.8	100	0.80
2	1.15	100	1.15	2	1.5	100	1.50
3	1.54	93.5	1.44	3	2.4	97.5	2.34
6	1.87	49	0.92	6	2.7	49	1.32
8	1.96	37	0.72	8	2.8	36.5	1.02
10	2.01	29.5	0.59	10	2.8	29.5	0.83
12	2.05	24.5	0.50	12	2.8	24.5	0.69
Pump submergence depth is 30%				Pump submergence depth is 30%			
1	0.49	100	0.49	1	0.9	100	0.90
2	0.83	83	0.69	2	1.4	94.5	1.32
3	1.00	60.5	0.61	3	1.8	64	1.15
6	1.21	31.5	0.38	6	2	32	0.64
8	1.27	24	0.31	8	2.1	23.5	0.49
10	1.30	19	0.25	10	2.1	19	0.40
12	1.33	16	0.21	12	2.1	15.5	0.33
Pump submergence depth is 20%				Pump submergence depth is 20%			
1	0.4	79.5	0.285				
2	0.6	57.5	0.331				
3	0.7	42	0.292				
6	0.8	22	0.186				
8	0.9	16.5	0.146				
10	0.9	13	0.118				
12	0.9	11	0.101				

submergence depth is 70%»), even though the pump submergence depth is greater than in the lower-pressure reservoir. In this case, as the stroke speed exceeds 3, the increase in daily production volume is accompanied by a sharp decrease in pump fillage. This behavior is due to the lower permeability of the reservoir. As a result, the maximum in the PPI versus stroke rate curve is observed (fig. 4).

In both reservoir pressures, at smaller pump submergence depths, the height of the fluid column in the annular space is significant, resulting in reduced depression and, consequently, a decrease in the inflow rate from the reservoir to the well. As a result, the PPI versus stroke speed curves for all other cases exhibit a maximum point. This behavior makes the PPI parameter practically interesting. Specifically, in sucker-rod pump operations, the maximum of PPI can be

used to select the optimal operating mode. In this context, the amplitude of PPI quantitatively reflects the pump's performance in that mode, with its range spanning from zero to the well's maximum possible daily production.

However, it should be noted that the PPI maximum does not always coincide with the maxima of daily production volume or pump fillage. In such cases, the PPI maximum allows determining the stroke speed that brings both parameters as close as possible to their maxima. This scenario is illustrated in figure 5, which shows the performance of a sucker-rod pump in a reservoir with a pressure of 12 atm and permeability of 160 mD, along with the PPI versus stroke speed curves. As seen from the graph, the maximum of the PPI curve occurs at a stroke speed *N*=2, whereas the pump fillage factor reaches its maximum at *N*=1, and the daily production

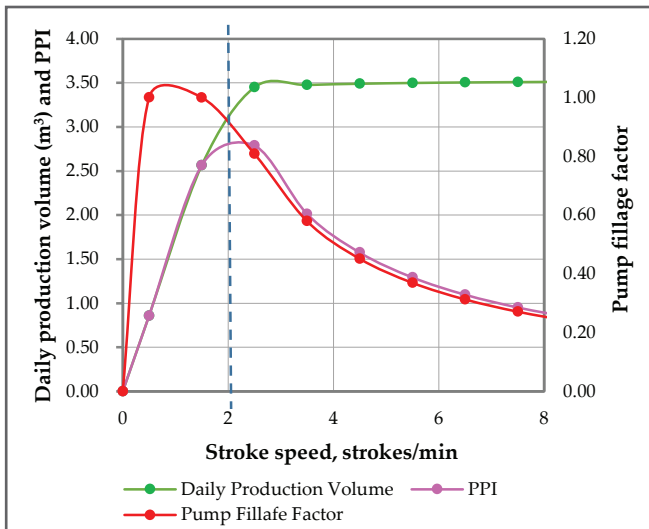


Fig. 5. The Pump Performance Indicator (PPI) objectively identifies the productivity-optimal operating regime under the considered conditions (reservoir pressure of 12 atm and permeability of 160 mD)

volume attains its maximum at $N=3$ strokes per minute. In this case, the PPI value is approximately 2.80. If the optimal stroke speed were determined based on the maximum pump fillage, the PPI would be approximately 1.5. If it were based on the maximum daily production volume, the PPI would be approximately 2.5.

This illustrates the practical significance of PPI, as it enables an objective assessment of the operating mode in a given case. While the PPI parameter is practically useful, it does not characterize the efficiency of the operating mode. PPI combines the actual volume of fluid produced by the pump with the pump’s production potential, but it does not account for the energy consumption in any explicit form.

To evaluate the technological efficiency of the production process, not only the volume of produced fluid but also the energy expended to achieve this result must be considered. In this context, it is necessary to extend Eq. (1) by incorporating a measurable parameter that accounts for energy consumption. In sucker-rod pumping systems, the electrical energy supplied to the system is directly related to the stroke rate of the beam pump. While an increase in stroke rate can lead to higher production, it is also accompanied by increased energy consumption, which may reduce overall efficiency. Therefore, including the stroke rate in Eq. (1) is both logical and physically justified for quantitatively expressing technological efficiency.

To achieve this, the theoretical volume of fluid lifted by the plunger during one full stroke (V_p) is used. This allows both non-dimensionalization of Eq. (1) and inclusion of the stroke speed (N) in the expression. Accordingly, the volume of fluid lifted per second during one stroke at a stroke speed N is:

$$V_p = \frac{N}{60} \pi r_p^2 l_p \quad (2)$$

where: N – stroke speed of the beam pump, strokes/min; r_p – radius of the pump cylinder, m; l_p – stroke length of the plunger, m.

To non-dimensionalize the Eq.(1), the ratio V/V_p can be used. It should be noted that V represents the volume of

fluid produced over a day (i.e., 86400 s). Then, Eq. (1) for the Technological Efficiency Coefficient (TEC) can be written as:

$$TEC = \frac{V}{86400V_p} \frac{F}{100} \quad (3)$$

Taking Eq. (2) into account, it can be expressed as:

$$TEC = \frac{60V}{86400N\pi r_p^2 l_p} \frac{F}{100} \quad (4)$$

The proposed expression considers both production indicators and energy expenditure simultaneously, enabling a more comprehensive and objective assessment of the technological efficiency of sucker-rod pumping operations (analysis of supporting research results is provided below). Eq. (4) has several significant advantages: it is dimensionless, and its range is limited to 0–1, making it universal and convenient for comparison under different operating conditions.

Eq. (4) integrates the pump’s potential output (through cylinder radius and plunger stroke length), the energy consumption regime (through the beam pump stroke speed), and the directly measurable actual performance indicators during operation (daily production volume and pump fillage) within a single mathematical framework. The quantity defined based on this expression is referred to as the Technological Efficiency Coefficient of the production process.

4. Simulation-based investigation of productivity and technological efficiency

To investigate the TEC indicator defined by Eq. (4), extensive computer simulations were performed. The obtained results were systematically analyzed and generalized. For this purpose, PPI and TEC values were calculated by varying the stroke speed from 0 to 10 strokes/min in increments of 0.5 for a low-productivity reservoir (reservoir pressure $P=60$ atm, drainage-zone permeability $k=16$ mD) and for relatively higher-productivity reservoirs ($P=100$ atm, $k=16$ mD and 32 mD). In all considered scenarios, the pump setting depth was kept constant. Such a selection of the study domain covers all possible cases from both geological and technological perspectives. The simulation results are presented in table 3 and figures 6–8. These results make it possible to observe the relationship between TEC and PPI under different reservoir and technical conditions, as well as their role in process identification.

The TEC and PPI curves shown in figures 6–8 (yellow and purple curves, respectively) indicate that in all scenarios the initial increase in stroke speed leads to an increase in PPI, i.e., pump productivity. This is expected and was discussed earlier. During this phase, TEC also increases sharply; however, after a certain value of N , TEC reaches its maximum and then remains nearly constant despite further increases in N (clearly visible in figs. 7 and 8). As reservoir productivity increases, the range of N over which TEC remains at its maximum also widens. Beyond a certain value of N , TEC begins to decline, and at this same value PPI attains its maximum. Thus, this value of N simultaneously ensures both maximum TEC and maximum PPI. Further increases in N beyond this point produce almost no increase in daily production. For example, in the relatively low-productivity reservoir (fig. 6), increasing N from 1.0 to 1.5 strokes/min raises daily production by only 1.3%, while at higher N values daily production remains essentially unchanged.

Table 3

Variation of the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC) as functions of the stroke speed under different reservoir conditions

Stroke speed, strokes/min	Daily production volume, m ³	Pump fillage, %	PPI	TEC
P=60 atm, k=16 mD				
0.00	0.0000	0.0000	0.0000	0.0000
0.5	0.7025	88.78	0.6237	0.7184
1.00	0.7376	44.4200	0.3276	0.1887
1.5	0.7491	29.6250	0.2219	0.0852
2.00	0.7550	22.2216	0.1678	0.0483
3.00	0.7605	14.816	0.1127	0.0216
4.00	0.7605	11.1132	0.0845	0.0122
5.00	0.7605	8.8911	0.0676	0.0078
6.00	0.7605	7.41	0.0564	0.0054
7.00	0.7605	6.349	0.0483	0.0040
8.00	0.7605	5.5573	0.0423	0.0035
9.00	0.7605	4.94	0.0376	0.0027
10.00	0.7605	4.4457	0.0338	0.0022
P=100 atm, k=16 mD				
0	0.0000	0.0000	0.3953	0.0000
0.5	0.3953	100.0000	0.7881	1.0000
1	0.7881	100.0000	1.1724	0.9999
1.5	1.1724	100.0000	1.4426	0.9999
2	1.4943	96.5416	1.1660	0.4154
2.5	1.5094	77.2482	0.9777	0.2686
3	1.5186	64.3817	0.7394	0.1877
4	1.5310	48.2951	0.5941	0.1065
5	1.5376	38.6386	0.4964	0.0684
6	1.5417	32.2010	0.4266	0.0477
7	1.5449	27.6114	0.3737	0.0351
8	1.5470	24.1539	0.3323	0.0269
9	1.5485	21.4611	0.2995	0.0213
10	1.5499	19.3236	0.3953	0.0172
P=100 atm, k=32 mD				
0	0.0000	0.0000	0.0000	0.0000
0.5	0.4009	100.0000	0.4009	1.0000
1	0.8015	100.0000	0.8015	0.9999
1.5	1.2001	100.0000	1.2001	0.9999
2	1.5990	100.0000	1.5990	0.9999
2.5	1.9964	100.0000	1.9964	0.9999
3	2.3901	100.0000	2.3901	0.9999
4	2.9763	93.9020	2.7948	0.4024
6	2.9978	62.6181	1.8772	0.1802
7	3.0042	53.6956	1.6131	0.1327
8	3.0082	46.9728	1.4130	0.1017
9	3.0112	41.7370	1.2568	0.0804
10	3.0140	37.5804	1.1327	0.0652

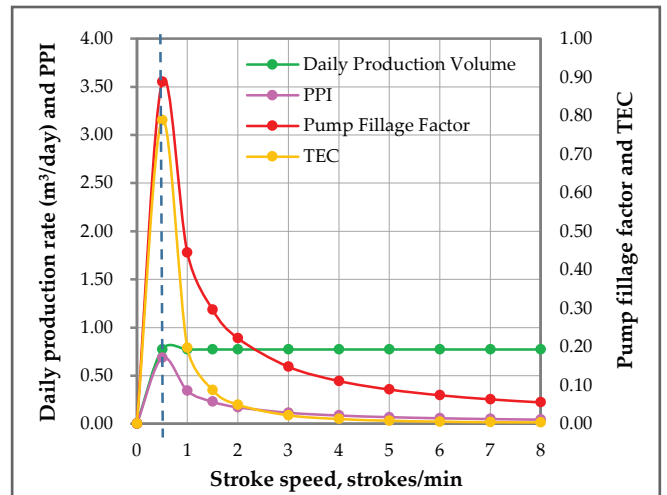


Fig. 6. Curves of the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC) as functions of the stroke rate, for a reservoir pressure of 60 atm and a drainage-zone permeability of 16 mD

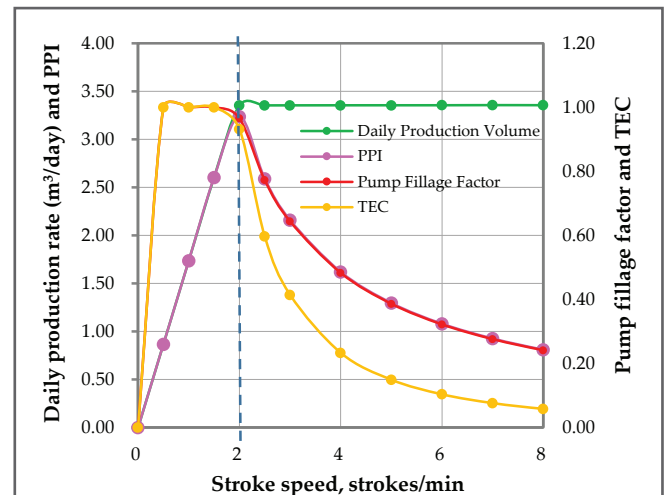


Fig. 7. Curves of the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC) as functions of the stroke rate, for a reservoir pressure of 100 atm and a drainage-zone permeability of 16 mD

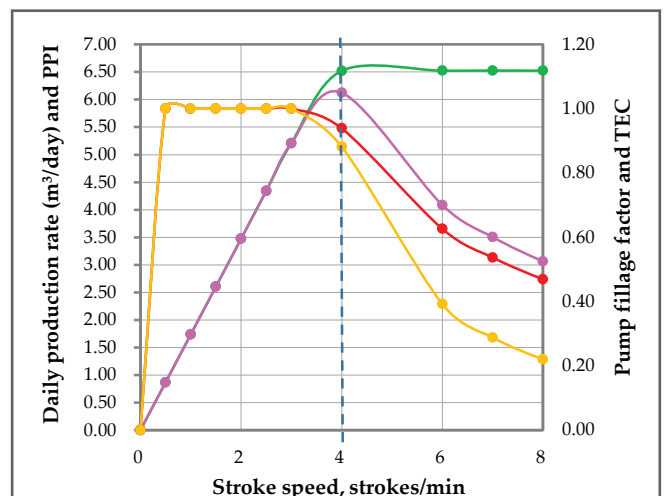


Fig. 8. Curves of the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC) as functions of the stroke rate, for a reservoir pressure of 60 atm and a drainage-zone permeability of 32 mD

This can also be stated as follows: for the considered well, there exists a minimum stroke speed at which PPI reaches its maximum while TEC begins to decrease. This stroke speed should be regarded as the optimal operating regime for the given scenario. The corresponding TEC value objectively represents the maximum technological efficiency of the regime for that well. For instance, in the reservoir with pressure 60 atm, the maximum TEC is 0.72, while the maximum pump performance indicator (PPI) equals 0.62.

A similar pattern is observed for reservoirs with relatively higher productivity (figs. 7 and 8). Higher reservoir pressure (100 atm) and, in addition, a twofold increase in permeability (figs. 7 and 8, respectively) both contribute to increased reservoir productivity. As a result, TEC remains constant up to $N \approx 2$ in the first case (fig. 7) and up to $N \approx 4$ in the second case (fig. 8). Within these intervals, PPI (purple curve) increases approximately linearly.

It is noteworthy that a comparison of reservoirs with twofold differences in permeability (16 and 32 mD, figs. 7 and 8) shows that although the higher permeability doubles daily production, TEC takes nearly identical values in both cases—approximately 0.45. This is expected, since the doubling of daily production is achieved by doubling the stroke speed. Therefore, the efficiency indicator remains the same in both cases. This fact further confirms the validity of the adopted TEC definition.

Another important observation is that the low-pressure reservoir ($P=60$ atm, $k=16$ mD), despite having productivity (PPI=0.62) that is 2.3 and 4.5 times lower than that of the higher-productivity reservoirs, exhibits higher technological efficiency: TEC=0.72. For comparison, in reservoirs with pressure 100 atm and permeability 16 mD and 32 mD, TEC equals 0.45 and 0.40, respectively. As seen, PPI adequately reflects regime productivity, whereas TEC adequately characterizes the efficiency of the production process.

The presented curves clearly show that optimizing sucker-rod pumping operation consists in determining the stroke speed that provides a compromise point between efficiency and productivity. At the same time, these results further confirm the practical significance of the proposed PPI and TEC parameters. Notably, the dependence of PPI on stroke speed qualitatively resembles the daily production curve of the pump, whereas the TEC curve resembles the pump fillage curve. This is natural, since by their physical meaning PPI characterizes volumetric productivity, while TEC characterizes efficiency.

It should be emphasized that PPI and TEC, for the first time, enable quantitative evaluation of both productivity and technological efficiency in sucker-rod pumping operations and serve as criteria for broader monitoring and optimization of the production process. TEC allows wells to be compared in terms of efficiency even when significant geological and technological differences exist between them, which gives it substantial practical importance. The proposed PPI and TEC indicators can be successfully applied not only in field operations but also as optimization criteria in computer simulation. The implementation of this concept in the X-Oil Laboratory simulator confirms this capability⁴.

5. Field application and operating-regime identification

The proposed technological efficiency concept was applied to determine the optimal operating regime and to evaluate the technological efficiency coefficient for Well No. 1220 in the Bibiheybat field. The current parameters of the well are as follows:

Static fluid level, m	396
Static liquid column height, m	134.1
Reservoir pressure, atm	11.2
Dynamic fluid level, m	512
Casing inner diameter, mm	255.1
Tubing diameter, mm	73
Pump setting depth (below static level), m	118
Plunger stroke length, mm	1500
Actual stroke rate, strokes/min	4
Pump cylinder diameter, mm	32
Permeability of the well drainage zone, mD	16
Actual pump cylinder fillage, %	7
Actual daily liquid production rate of the well, m ³ /day	0.490

Based on these input data, the production process was simulated, and the daily produced fluid volume and the pump fillage were predicted for various stroke speeds. Using the obtained results, the values of PPI and TEC were calculated according to Eqs. (1) and (4), respectively. All calculations were performed for stroke speed ranging from 1 to 8 strokes per minute. The calculation results are presented in table 4.

As can be seen from the table 4, the maxima of both PPI and TEC correspond to a stroke rate of 1 stroke per minute. At this stroke speed, PPI and TEC reach their maximum values. Increasing the stroke rate above 1 leads to a decrease in both PPI and TEC, while the daily production volume remains nearly unchanged. This can be observed in figure 9 from the curves showing the dependence of PPI, TEC, and daily production volume on the stroke speed N (purple, yellow, and green curves, respectively).

At $N=1$, the value of TEC is 0.08, which is significantly lower than its theoretical maximum of 1. In this regime, PPI is equal to 0.14. These values are natural for a well with such a low production rate of 0.48 m³/day. The $N=1$ regime is optimal, as both the efficiency coefficient and the productivity indicator attain their maximum values under these conditions. However, in order to increase production, the well is currently operated at a stroke speed of $N=4$ (green row in table 4). According to the TEC versus stroke speed curve, in this regime TEC decreases to a value close to zero (0.0051), while PPI drops to 0.0357. Increasing N fourfold results in only a 3.71% increase in daily production, which corresponds to merely 0.018 m³/day of additional oil. In making such a decision, the operating company can evaluate the associated economic risks. In particular, when operating at $N=4$, the increase in energy consumption, the reduction in the mean time between repairs, and other additional costs should be assessed relative to operation at $N=1$, and compared with the revenue obtained from the additional 0.018 m³/day of oil production achieved at $N=4$.

⁴ A tool has been developed in the X-Oil Laboratory simulator to automatically determine the optimal regime based precisely on these parameters.

Table 4

Variation of production parameters, the Pump Performance Indicator (PPI), and the Technological Efficiency Coefficient (TEC) with stroke speed for Well No. 1220 of the Bibiheybat field

Stroke Speed, strokes/min	Daily Production Volume, m ³	Pump Fillage, %	PPI	TEC
0	0	0	0	0
1	0.4773	28.7469	0.1372	0.0790
2	0.4892	14.4002	0.0704	0.0203
3	0.4931	9.6060	0.0474	0.0091
4	0.4950	7.2062	0.0357	0.0051
5	0.4961	5.7664	0.0286	0.0033
6	0.4968	4.8059	0.0239	0.0023
7	0.4973	4.1181	0.0205	0.0017
8	0.4976	3.6047	0.0179	0.0013

Note: The green row represents the actual operating parameters

This example clearly demonstrates that the PPI and TEC parameters function as intended, correctly identifying the productivity and efficiency of the operating regime.

6. Advantages of the proposed PPI-TEC framework over conventional performance indicators

The performance of sucker-rod pumping systems is traditionally evaluated using production rate, pump fillage factor, or volumetric efficiency. In conventional practice, the latter is commonly defined as the ratio of the actual daily production to the theoretical pump capacity calculated under the assumption of complete cylinder filling at the specified stroke length and stroke speed. Consequently, this efficiency measure primarily reflects the degree of pump filling and provides limited information about the overall technological performance of the production system.

Although such indicators are useful for assessing individual aspects of pump operation, they do not explicitly describe the relationship between productivity and technological efficiency or support the selection of optimal operating regimes. In particular, they cannot reveal situations in which increasing stroke speed produces only marginal production gains while causing a substantial deterioration in operating efficiency.

The proposed framework addresses these limitations

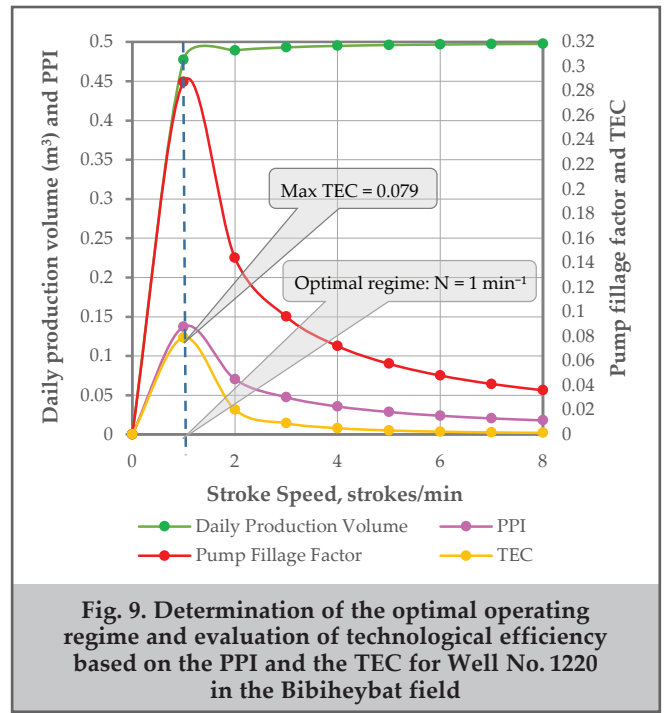


Fig. 9. Determination of the optimal operating regime and evaluation of technological efficiency based on the PPI and the TEC for Well No. 1220 in the Bibiheybat field

through the combined use of the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC). PPI characterizes production performance by integrating actual production with pump fillage, whereas TEC evaluates technological efficiency by simultaneously accounting for production, pump fillage, and operating intensity. Considered individually, each indicator provides only a partial description of system behavior. However, their combined interpretation offers a comprehensive characterization of the productivity–efficiency trade-off that governs sucker-rod pumping operation.

An important consequence of this approach is that operating regimes can be evaluated from a multi-criteria perspective rather than solely on the basis of production maximization. The results presented in this study demonstrate that the maxima of PPI and TEC generally occur at different stroke speeds, implying that rational operating conditions should be selected as a compromise between productivity and technological efficiency. This capability represents a significant advantage over conventional evaluation methods and provides a more informative basis for engineering decision-making and production optimization. The novelty of the proposed approach lies not only in the definition of the PPI and TEC indicators themselves, but primarily in their combined interpretation as a framework for multi-criteria operating-regime selection.

Conclusions

- Large-scale computer experiments were conducted for various reservoir conditions and operating regimes using the X-Oil Laboratory simulation platform developed on the basis of the authors’ Discrete-Imitation Modeling Concept. Based on these simulations, two integral indicators—the Pump Performance Indicator (PPI) and the Technological Efficiency Coefficient (TEC)—were formally defined, and their dependence on directly measurable field parameters was extensively investigated.

- The presence of an extremum in the functional relationship between these indicators and energy-demand-related parameters, such as stroke speed, makes them particularly suitable for evaluating and optimizing the productivity and efficiency of sucker-rod pumping operations. The PPI- and TEC-based methodology captures the dynamic processes occurring within the pump–well–reservoir system and enables the assessment of both pump performance and overall operational efficiency under real operating conditions.
- Within this framework, criteria are introduced for the first time that allow the quantitative evaluation of both productivity and technological efficiency in sucker-rod pumping operations, as well as broader monitoring and optimization of the production process. In particular, the TEC parameter enables, for the first time, a consistent comparison of wells in terms of efficiency, even when significant geological and technological differences exist among them. For this reason, TEC has substantial practical significance.
- The results indicate that, under the considered operating conditions, the PPI reflects the productivity of the operating regime, whereas the TEC objectively evaluates the technological efficiency associated with that productivity. Most importantly, it enables comparison across different wells and operating scenarios. If an operating company is interested in achieving a slight increase in daily production at the expense of deteriorating PPI and TEC values, this decision becomes a matter of production policy. In such cases, the economic risk associated with the additional energy consumption caused by increased stroke speed can be estimated through straightforward calculations.
- Both indicators are sensitive to key operational parameters, including pump characteristics, PVT and rheological properties of reservoir fluids, and reservoir conditions, confirming their reliability as analytical tools for evaluating and optimizing operating regimes.
- The proposed PPI and TEC indicators can be effectively used not only in field applications but also as optimization criteria in computer simulations. This enables the simulator to automatically determine operating regimes corresponding to optimal or specified TEC and PPI values, as confirmed by the authors' experience with the X-Oil Laboratory simulator.
- This study also demonstrates a methodological approach for investigating sucker-rod pumping wells using simulation tools.
- The proposed methodology was applied to a real production case—Well No. 1220 of the Bibiheybat field—where its practical applicability and functional robustness were successfully validated.
- Future research will focus on extending and adapting the proposed methodology to intermittent operating regimes of sucker-rod pumping systems.

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