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A GENERALIZED HYDRAULIC CALCULATION MODEL FOR NON-NEWTONIAN FLUID PIPE FLOW AND ITS APPLICATION EVALUATION

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A generalized hydraulic model that is independent of the rheological model is presented for non-Newtonian fluids flow in pipes. The generalized model was developed without assuming that the generalized flow index (n') remains constant over all shear rates. Based on the pipe flow control equation, the explicit equations between the wall shear stress and volumetric flow of all common rheological models flowing in pipe were obtained, such as the Casson model, Herschel-Bulkley model and the Robertson-Stiff model, and they all can be solved numerically to obtain the accurate wall shear rate and shear stress. We give the theoretic calculation method of n' for all time-independent non-Newtonian fluids. This method is applicable to all common rheological models without the assumption that n' is constant. Moreover, we derived the general expression of generalized Reynolds number and then gave a utility pressure loss calculation model. A set of measured hydraulic data were utilized to evaluate this model. The results show that it can precisely calculate pipe flow parameters for all types of different non-Newtonian fluid. The proposed model will lay a foundation for the application and more extensive use of complex but more precise rheological models in engineering.

Keywords: pipe flow, hydraulic calculation, generalized Reynolds number, evaluation analysis.

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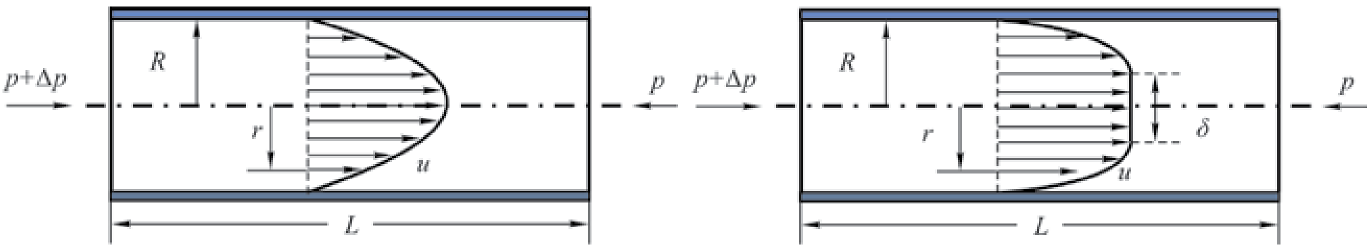
Introduction

The study of non-Newtonian fluid flow in pipes have important applications in many industrial fields including petroleum drilling, oil & gas storage and transportation, and chemical production. Take the drilling as an example, with the increase of slim hole, complex structure wells and the drilling depth, the requirement of hydraulic calculation accuracy is more and more high, especially with the implementation and promotion of management pressure drilling technology in the narrow density window formation [1-3]. Because the annular conditions of the drilling engineering are complex and annular pressure loss is influenced by many factors, the annular hydraulic calculation accuracy is difficult to meet the requirements. But we could improve the precision of the hydraulic parameters in drilling pipes and then calculate the pressure loss in annular used the standpipe pressure based on the U-tube principle.

There are many publications in the literature that deal with the flow of non-Newtonian fluids in pipes. The Bingham (BH) plastic [4] and power law [5] (PL) are often used for non-Newtonian fluid pipe hydraulic calculation because of their simplicity and their description of fluid rheology. The American Petroleum Institute standard discussed these pipe hydraulic models. In reality, most drilling fluids correspond much more closely to yield pseudoplastic fluids, so some more complex rheological models were proposed to describe their rheological behavior and estimate hydraulic parameters. For example, Herschel & Bulkley [6] (HB) presented a yield power law model and the cementing research section of Southwest Petroleum Institute (China) [7] published their papers to discuss the application of this model. Casson [8] (CS) presented an improved two-parameter model and its application was discussed by Chen and Liu [9]. Also, Robertson and Stiff [10] (RS) presented a new rheological model and the application of RS model was described in detail by Liu and Huang [11,12]. In order to obtain the friction pressure loss equation, which is a function of geometry, flow rate, density and rheological parameters, most of the above models omitted some calculation items.

A reasonably general relationship between the steady, stabilized, laminar flow of any time-independent purely viscous fluid (with no yield stress, such as PL fluid) may be developed from a generalized constitutive equation, which called pipe generalized flow rate equation and was developed by Herzog & Weissenberg [13]. Based on the generalized flow rate equation, Metzner & Reed [14] introduced n' for PL fluid flow in pipe, which was called generalized flow index, and suggested to measure its value by a capillary-tube viscometer. Based on the Metzner & Reed's approach, several scholars have done some research for other common rheological models [15,16]. But all of them based on the premise that n' is constant over all range of shear rates, and didn't give a simple calculation method for n'. Recently, Haobo et al. described the hydraulic calculation method of four parameter [17] (FP)



a) pseudoplastic fluids **b) yield pseudoplastic fluids**

Fig.1. Illustration of non-Newtonian fluid flow in pipe

rheological model based on the Metzner & Reed's approach, but they didn't generalize this method to the other common rheological models [18]. And so in this article a generalized hydraulic calculation for non-Newtonian fluid in pipes was proposed, which was applicable for all common rheological model and was developed without assuming that the generalized flow index (n') remains constant over all shear rates, and then the theoretic calculation method of n' and the utility hydraulic method for all time-independent non-Newtonian fluid were proposed.

1. Generalized flow equation of pipe flow

The geometry of non-Newtonian fluid pipe flow is depicted in figure 1, where R is the radius of pipe. From the figure we can see that there will be a central core that moves like a rigid plug when shear stress levels are smaller than the yield stress of the fluid for yield pseudoplastic fluids and the thickness of central core is δ, this is not the case for the fluids with no yield stress, such as PL and Sisko [19] fluid.

And as for the steady state laminar flow of any time-independent purely viscous fluid in pipe, Herzog & Weissenberg first developed an equation called pipe flow rate equation that relate the pipe flow rate to the wall shear stress [13]:

$$Q = \frac{\pi R^3}{\tau_w^3}\int_0^{\tau_w} \tau^2 \gamma d\tau \tag{1}$$

Where Q is flow rate, m³/s;

R is the radius of pipe, m;

τ_w is the wall shear stress, Pa.

The eq.(1) is generalized flow rate equation for the purely viscous fluid flowing in pipe, such as PL fluid and Sisko fluid. But the eq.(1) can't describe the flow rate of yield pseudoplastic fluids because there will be a central core. And so we should get the flow rate equations for yield pseudoplastic fluids firstly. Based on the simple force balance principle, the shear stress of the fluid layer at r can be described as [20]:

$$\tau = \frac{\Delta P r}{2L} \tag{2}$$

Where τ is shear stress, Pa;

ΔP is the pressure loss, Pa;

L is the length of pipe, m.

And then the wall shear stress can be expressed as:

$$\tau_w = \frac{\Delta P R}{2L} \tag{3}$$

For the steady flow in pipe, assuming the velocity is u at r. the flow rate equation can be expressed as:

$$Q = 2\pi\int_0^R ur\mathrm{d}r = u\pi r^2\Big|_0^R + \int_0^R \pi r^2\left(-\frac{\mathrm{d}u}{\mathrm{d}r}\right)\mathrm{d}r \tag{4}$$

Based on the principle that no slip at the wall, so $u = 0$ when $r = R$, this results in $u\pi r^2\Big|_0^R = 0$. Therefore eq.(4) can be simplified to:

$$Q = 2\pi\int_0^R ur\mathrm{d}r = \pi\int_0^R r^2\left(-\frac{\mathrm{d}u}{\mathrm{d}r}\right)\mathrm{d}r \tag{5}$$

And for yield fluids such as H-B, R-S fluids, etc. since $-du/dr = 0$ when $r < \delta/2$, so eq.5 may be expressed as:

$$Q = 2\pi\int_0^R ur\mathrm{d}r = \pi\int_{\frac{\delta}{2}}^R r^2\left(-\frac{\mathrm{d}u}{\mathrm{d}r}\right)\mathrm{d}r \tag{6}$$

Combining eq.(2) and eq.(3) and rearranging them the following expressing was obtained:

$$r = \frac{R}{\tau_w}\tau \quad ; \quad \mathrm{d}r = \frac{R}{\tau_w}\mathrm{d}\tau \tag{7}$$

In order to solve eq.(6), we utilized the fact that $\gamma = -du/dr = f(\tau)$, then we substituted eq.(7) back into eq.(6) and we obtained:

$$Q = \frac{\pi R^3}{\tau_w^3}\int_{\tau_0}^{\tau_w} \tau^2 \gamma \mathrm{d}\tau \tag{8}$$

Obviously, eq.(1) is the generalized pipe volumetric flow rate formula for pseudoplastic fluids such as P-L and Sisko fluids, while eq.(8) describes yield pseudoplastic fluids, such as H-B fluids, R-S fluids, etc. We can substitute the rheological model equation into eq.(1) or eq.(8) and integrate them, and then the respective concrete flow rate equation can be derived.

Table 1 gives the flow rate equations obtained from both the proposed model and the traditional hydraulic model [7,9,11-12,20]. From table 1 we can see that the flow rate equation obtained from the proposed method were more accuracy because of the flow rate of tradition hydraulic model have omitted some calculation items, such as the $\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^4$ was omitted for BH model, the $\left(\frac{\tau_0}{\tau_w}\right)^4$, $\left[1-\left(\frac{\tau_0}{\tau_w}\right)^{3.5}\right]$ and $\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)\left[1-\left(\frac{\tau_0}{\tau_w}\right)^3\right]$ were omitted for CS model, et al. From table 1 we can also find that the flow rate equation of Sisko and FP model can't be obtained based on the traditional hydraulic model, but this is not the case for the proposed model. For a given flow rate, and

Table 1

The flow rate equation of proposed model and traditional hydraulic model

Rheological model		Flow rate equation	
Name	Equation	The traditional method	The proposed method
BH	$\tau=\tau_0+\mu_p\gamma$	$Q=\frac{\pi R^3\tau_w}{4\mu_p}\left[1-\frac{4}{3}\left(\frac{\tau_0}{\tau_w}\right)\right]$	$Q=\frac{\pi R^3\tau_w}{4\mu_p}\left[1-\frac{4}{3}\left(\frac{\tau_0}{\tau_w}\right)+\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)^4\right]$
PL	$\tau=K\gamma^n$	$Q=\frac{n\pi R^3}{(3n+1)}\left(\frac{\tau_w}{K}\right)^{\frac{1}{n}}$	$Q=\frac{n\pi R^3}{(3n+1)}\left(\frac{\tau_w}{K}\right)^{\frac{1}{n}}$
CS	$\tau^{0.5}=\tau_0^{0.5}+\eta_\infty^{0.5}\gamma^{0.5}$	$Q=\frac{\pi R^3\tau_w}{\eta_\infty}\left\{\frac{1}{4}-\frac{4}{7}\left(\frac{\tau_0}{\tau_w}\right)^{0.5}+\frac{1}{3}\frac{\tau_0}{\tau_w}\right\}$	$Q=\frac{\pi R^3\tau_w}{\eta_\infty}\left\{\frac{1}{4}\left[1-\left(\frac{\tau_0}{\tau_w}\right)^4\right]-\frac{4}{7}\left(\frac{\tau_0}{\tau_w}\right)^{0.5}\left[1-\left(\frac{\tau_0}{\tau_w}\right)^{3.5}\right]+\frac{1}{3}\left(\frac{\tau_0}{\tau_w}\right)\left[1-\left(\frac{\tau_0}{\tau_w}\right)^3\right]\right\}$
HB	$\tau=\tau_0+K\gamma^n$	$Q=\frac{n\pi R^3}{3n+1}\left(\frac{\tau_w}{K}\right)^{\frac{1}{n}}\left[1-\frac{3n+1}{n(2n+1)}\frac{\tau_0}{\tau_w}\right]$	$Q=\frac{n\pi R^3}{3n+1}\left(\frac{\tau_w}{K}\right)^{\frac{1}{n}}\left(1-\frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}}\left[1+\frac{2n}{2n+1}\left(\frac{\tau_0}{\tau_w}\right)+\frac{2n^2}{(n+1)(2n+1)}\left(\frac{\tau_0}{\tau_w}\right)^2\right]$
RS*	$\tau=A(\gamma+C)^B$	$Q=\frac{\pi B}{3B+1}\left(\frac{\tau_w}{A}\right)^{\frac{1}{B}}R^3-\frac{\pi C}{3}R^3$	$Q=\frac{\pi BR^3}{3B+1}\left(\frac{\tau_w}{A}\right)^{\frac{1}{B}}\left[1-\left(\frac{\tau_0}{\tau_w}\right)^{\frac{3B+1}{B}}\right]-\frac{\pi CR^3}{3}\left[1-\left(\frac{\tau_0}{\tau_w}\right)^3\right]$
Sisko	$\tau=a\gamma+b\gamma^n$	— — —	$Q=\frac{\pi R^3}{\tau_w^3}\left(\frac{1}{4}a^3\gamma_w^4+a^2b\frac{n+2}{n+3}\gamma_w^{n+3}+ab^2\frac{2n+1}{2n+2}\gamma_w^{2n+2}+b^3\frac{n}{3n+1}\gamma_w^{3n+1}\right)$
FP[18]	$\tau=\tau_0+a\gamma+b\gamma^c$	— — —	$Q=\frac{\pi R^3}{\tau_w^3}\left(\frac{1}{2}a\tau_0^2\gamma_w^2+\frac{2}{3}a^2\tau_0\gamma_w^3+\frac{1}{4}a^3\gamma_w^4+\frac{bc}{c+1}\tau_0^2\gamma_w^{c+1}+2ab\frac{c+1}{c+2}\tau_0\gamma_w^{c+2}\right.$ $\left.+a^2b\frac{c+2}{c+3}\gamma_w^{c+3}+\frac{2b^2c}{2c+1}\tau_0\gamma_w^{2c+1}+ab^2\frac{2c+1}{2c+2}\gamma_w^{2c+2}+b^3\frac{c}{3c+1}\gamma_w^{3c+1}\right)$

*) The yield stress of RS model τ_0=AC^B.

with rheological equation and the concrete flow rate equation, the values of wall shear rate γ_w and shear stress τ_w can be obtained using numerical methods.

2. Generalized hydraulic calculation model

2.1 Calculation model of Generalized flow index

2.1.1 Generalized flow index

Rabinowitsch [21] & Mooney [22] introduced the pipe flow characteristic parameter (8v/D, which is also called Newtonian wall shear rate of pipe flow, viz. $\gamma_{w'n}$) based on eq.(1), which gives a general relationship for the shear rate at the wall for the pseudoplastic fluids. And then Metzner & Reed [14] defined a generalized flow index for Power-Law fluid.

Following Metzner & Reed we obtained the generalized flow index for yield-pseudoplastic fluids. Assuming the mean velocity is v, the volumetric flow rate of pipe flow can also be written as:

$$Q=\pi R^2 v \tag{9}$$

Combining eq.(8) and eq.(9), the following equations can be derived:

$$\frac{8v}{D}=\frac{4}{\tau_w^3}\int_{\tau_0}^{\tau_w}\tau^2\gamma\,\mathrm{d}\tau \tag{10}$$

Eq.(10) is a perfectly general relationship of the mean velocity v, he wall shear stress τ_w, the diameter D, and the shear rate γ. Then we can take a derivative with respect to τ_w on both sides of eq.(10), and then the following equation can be obtained:

$$\tau_w^3\frac{d(8v/D)}{d\tau_w}+3\tau_w^2\frac{8v}{D}=4\tau_w^2\gamma_w \tag{11}$$

And the eq.(11) may be rearranged as following:

$$\gamma_w=f(\tau_w)=\frac{3}{4}\left(\frac{8v}{D}\right)+\frac{1}{4}\tau_w\frac{d(8v/D)}{d\tau_w} \tag{12a}$$

$$\gamma_w=f(\tau_w)=\frac{3}{4}\left(\frac{8v}{D}\right)+\frac{1}{4}\frac{\left[d(8v/D)/(8v/D)\right]}{(d\tau_w/\tau_w)}\left(\frac{8v}{D}\right) \tag{12b}$$

Since $(d\tau_w/\tau_w)=d\ln\tau_w$ and $[d(8v/D)/(8v/D)]=d\ln(8v/D)$, and so the eq.(12b) can be rearranged as following:

$$\gamma_w=f(\tau_w)=\frac{3}{4}\left(\frac{8v}{D}\right)+\frac{1}{4}\frac{d\ln(8v/D)}{d\ln\tau_w}\left(\frac{8v}{D}\right) \tag{13}$$

Eq.(13) is the wall shear rate expression for yield pseudoplastic fluids in pipe, we were also able to define the generalized flow index for yield pseudoplastic fluids:

$$n'=\frac{\mathrm{d}\ln\tau_w}{\mathrm{d}\ln\left(8v/D\right)} \tag{14}$$

Eq.(14) is the generalized flow index for yield pseudoplastic fluids in pipe, which is the same formula as Metzner & Reed proposed for P-L fluid. Introducing the above expression to eq.(13) and simplifying, the following equation for γ_w can be obtained:

$$\gamma_w=\frac{3n'+1}{4n'}\left(\frac{8v}{D}\right) \tag{15}$$

2.1.2. Calculation model of n'

The parameter n' is a vital parameter, but the predecessors didn't give the calculation method for n'. Metzner & Reed [14] suggested to measure the value of n' by a capillary tube viscometer, but usually use the rotational viscometer measure the rheological properties of working fluid, such as in the drilling engineering. Khataniar et al. [16] developed a method for determination of flow regimes in pipe flow for the H-B and R-S model based on the determination of flow regime defined for Power law model, they used the rheological models' flow index to replace n' in the critical Reynolds number formula because there was no effective calculation of n' for these rheological model. Neither Metzner, his students, nor subsequent workers gave a simple and universal calculation model for n'. Therefore, we must obtain an effective and simple calculation model for n'.

Eq.(15) is the simplified formula of wall shear rate for both of pseudoplastic and yield pseudoplastic fluids in pipes, which is a function of the generalized flow index n' and the Newtonian wall shear rate, viz. $8v/D$, if the flow rate and wall shear rate is known, we could calculate the generalized flow index n' easily.

To summarize, we established a utility calculation model for n' and the flow diagram is given in figure 2. The current approach uses a bisection method. Two values of γ_w are assumed, where γ_{w1} is set to $\gamma_{w'n'}$ and γ_{w2} is set to a very high value of $10\gamma_{w'n}$. The γ_{w0} is then the arithmetic mean of the two. The flow rate Q for each shear rate can be computed based on the flow equation. The procedure converges when $|Q_0\text{-}Q_{given}|/Q_{given}<0.001\%$. And then the generalized flow index n′ can be calculated easily based on the eq.(15).

The preceding paragraphs we give the calculation model of n', and it's applicable for both pseudoplastic and yield-pseudoplastic fluids flow in pipe and is independent on the rheological model. This simple calculation method provides the foundation for the application and popularization of generalized flow index hydraulic method.

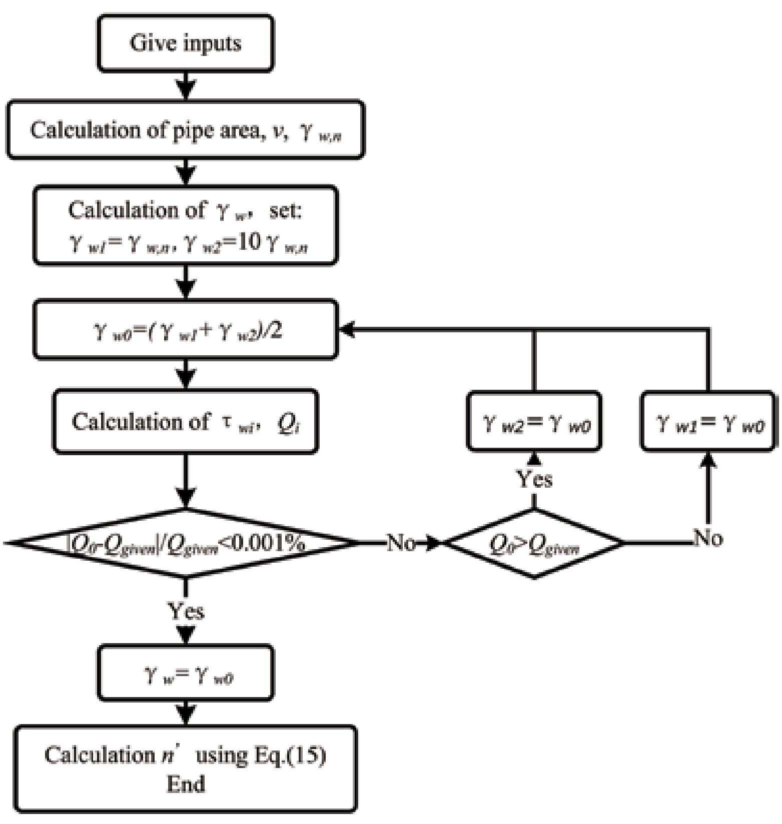


Fig.2. The flow diagram for generalized flow index calculation

2.2. *Generalized Reynolds number*

Reed & Pilehvari [15] defined an "effective diameter" so that the shear rate of non-Newtonian fluid can be put into the same formation as the wall shear rate for Newtonian pipe flow, the effective diameter for generalized non-Newtonian pipe flow is:

$$D_{eff} = \frac{4n'}{3n'+1}D \tag{16}$$

And then they defined the apparent viscosity for the fluid that n' is constant, viz. PL fluid, $\mu_{w,app}=K'(8v/D)^{n'}/\gamma_w$. Using this parameter they gave a generalized Reynolds number as following:

$$Re_g = \frac{\rho D_{eff} v}{K'\left(8v/D\right)^{n'}/\gamma_w} \tag{17}$$

The eq.(17) was established based on the condition that n' is a constant, and so it was only applicable for Power-law fluid flow in pipe.

Through theoretical analysis, we found that the generalized Reynolds number formula can be derived without the assumption that n' is constant. The Fanning friction factor for the steady, stabilized, laminar flow in pipe defined as follow:

$$f = \frac{2\tau_w}{\rho v^2} = \frac{16}{\dfrac{\rho D v}{\tau_w / \frac{8v}{D}}} \tag{18}$$

We can see that eq.(18) is valid for both of Newtonian and non-Newtonian fluid flow in pipes. Referenced to the formula relate friction factor to Reynolds number for Newtonian fluid laminar pipe flow, viz. f=16/Re, the generalized Reynolds number for any time-independent non-Newtonian fluid pipe flow can be obtained:

$$Re_g = \frac{\rho D v}{\tau_w / \frac{8v}{D}} \tag{19}$$

Combining eq.(15) and (19), the eq.(19) for generalized Reynolds number can be further expressed as following:

$$Re_g = \frac{\rho D v}{\left(\frac{3n'+1}{4n'}\right)\tau_w/\gamma_w} = \frac{\rho\left(\frac{4n'}{3n'+1}\right)Dv}{\tau_w/\gamma_w} = \frac{\rho D_{eff} v}{\tau_w/\gamma_w} = \frac{\rho D_{eff} v}{\mu_{w,app}} \tag{20}$$

As well be shown, above derivation for the Reg has nothing to do with whether n′ is a constant and that eq.(20) is applicable for both Newtonian and non-Newtonian fluid. Therefore, based on the generalized flow index n' and generalized Reynolds number Reg, we could extend the generalized flow index method to all time-independent non-Newtonian fluid pipe flow analysis and related non-Newtonian fluid to Newtonian fluid more easier.

2.3. *Regime transitions*

Metzner and Reed [14] first investigated the laminar-turbulent-transition criterion and determined the critical Reynolds number by plotting the friction factor of non-Newtonian fluids against the generalized Reynolds number under laminar conditions. They proposed that both Newtonian and non-Newtonian fluids leave the region of stable laminar conditions when f first drops to a value of about 0.008 or less or when Re_g reaches a value of 2000 to 2500. Dodge and Metzner [23] found that the transition from laminar to turbulent flow with non-Newtonian systems took place within a Reynolds number range comparable to the extent of the Newtonian transition region and they found the lower critical Reynolds number that corresponds to the onset of turbulent flow appeared to increase slightly as the values of the flow-behavior index decreased. Schuh [24] then calculated the critical Reynolds number defining the laminar to non-laminar flow transition of PL fluids, and the result was $(Re_g)_{cr}$ = 3470-1370n. Based on the proposed generalized flow index and generalized Reynolds number, the laminar to non-laminar transition critical Reg may be written as:

$$(Re_g)_{cr} = 3470-1370n' \tag{21}$$

Metnzer and Reed indicated that there are uncertainties relating to the actual width of the transition region [14]. Although Schuh designed their formulation to cover the fully turbulent transition boundary as $(Re_g)_{cr}$ = 4270-1370n, this model is less well substantiated in the literature than the laminar to non-laminar transition boundary. Therefore, in this paper we used eq.(21) to predict the lower critical Reynolds number and determine if the flow regime is laminar or turbulent. Furthermore, as in section 3, we will compare the results of these transition criteria against observed data for several fluid types in pipe.

2.4. *Pressure loss calculation*

The next step in the mathematical development is to relate the Reynolds number and the Fanning friction factor (f) after obtaining n', Re_g and D_{eff}. As mentioned previously, the laminar flow Fanning friction factor can be expressed as follows:

$$f = \frac{16}{Re_g} \tag{22}$$

The Fanning friction formula that was used to describe non-laminar flow while accounting for the transition region and fully turbulent flow is a combination of the Dodge-Metzner [23] equation and Colebrook's equation [25], which includes the roughness effect. The derivation was performed by Reed and Pilehvari and the formula follows [15]:

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left\{\frac{0.27\varepsilon}{D_{eff}} + \frac{1.26^{(n')^{-1.2}}}{\left[Re_g f^{(1-0.5n')}\right]^{n'^{-0.75}}}\right\} \tag{23}$$

Then, the pressure loss is calculated by the following equation:

$$\Delta P = 2f\frac{\rho v^2}{D}L \tag{24}$$

2.5. *Hydraulic calculation procedure*

To summarize, we established a utility hydraulic calculation model for non-Newtonian fluid flow in pipe. The flow diagram is given in figure 3. From the figure we can see the selection of rheological model and derivation of flow rate equation using eq.(1) or eq.(8) is the basis of hydraulic calculation. Based on the proposed calculation procedure to calculate the n'. Then the other hydraulic parameters can be easily obtained. Then a Visual Basic computer program was written to implement the computational procedure outlined here. The code is also being interfaced as ActiveX DLL to create a user-friendly program.

3. Results and discussion

The generalized hydraulic calculation model described above was tested using the experimental data of Subramanian, who measured pressure losses for a wide variety of drilling fluids in a large flow loop operated by AMOCO [26]. The conduit sections

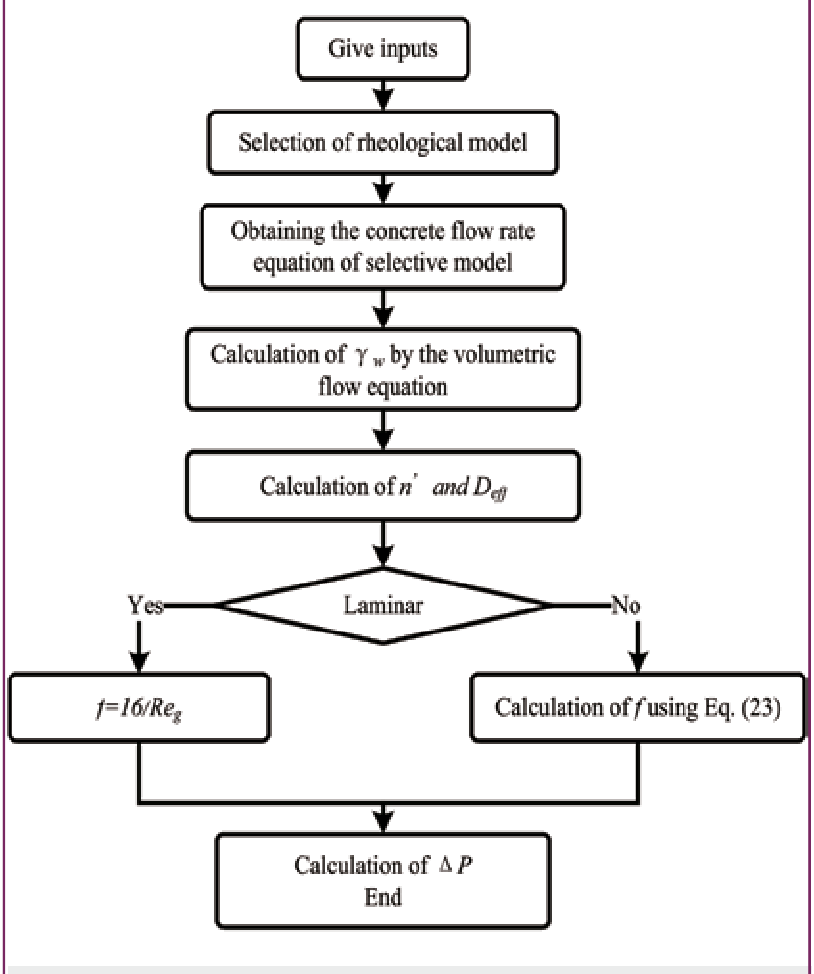


Fig.3. The flow diagram for hydraulics calculation

contained smooth and rough pipes, and these data were several different fluids flow in smooth and rough pipes, such as Bentonite mud, mixed-metal-hydroxide (MMH) mud and Glycol mud.

Figure 4 through figure 6 compare predicted and experimental pressure losses of three different Bentonite muds flow in smooth and rough pipes. The

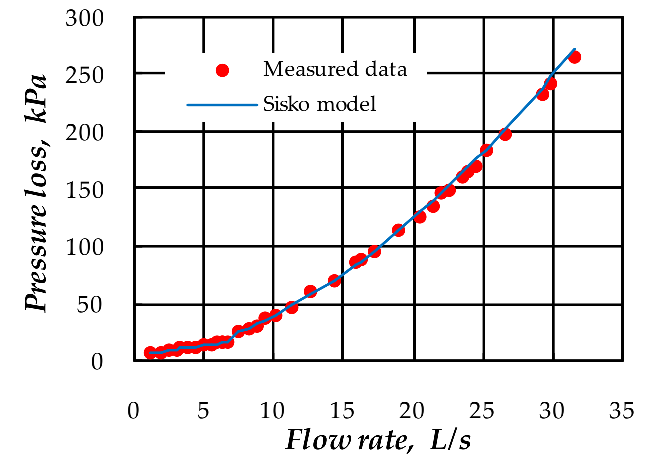


Fig.4. Comparison of predictions with measurements of Bentonite Mud 1 in smooth pipe (Sisko model: a=0.01507 Pa·s, b=1.13557 Pa·s^c, c=0.403)

yield stress of Bentonite mud 1 is very low and we use Sisko model to describe its property. The bentonite mud 2 and mud 3 have higher yield stress than mud 1, and so we used RS model to describe mud 2 and HB model for mud 3. Figure 5 also gave the predictions based on the traditional model and figure 6 gave the predictions that quoted from [26]. Because there is no traditional model for sisko model and so we gave the predictions based on the proposed model only. From the figures we can see there are excellent matches between the predictions of proposed model and the experimental results. And the predictions of proposed model were more accurate than the traditional models.

Moreover, we give the predictions of MMH mud and Glycol mud flow in pipes. For describing the proposed model is applicable for all common rheological model, we used CS model to describe the rheological property of MMH mud and FP model for Glycol mud. In the figure 7 we also give the predictions of traditional CS hydraulic model, the same to Sisko model, there is no traditional hydraulic for FP model and we give the predictions of proposed model. The figures also show excellent matches between the predictions based on the proposed model and experimental values.

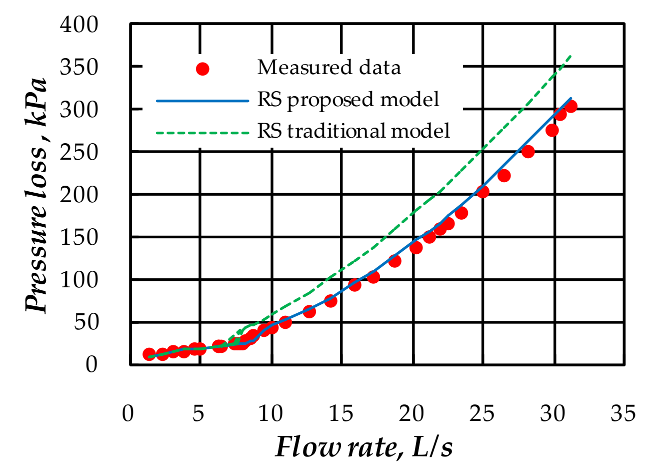


Fig.5. Comparison of predictions with measurements of Bentonite Mud 2 in rough pipe (RS model: A=0.83827 Pa·s^B, B=0.5707, C=4.60085 s^{-1})

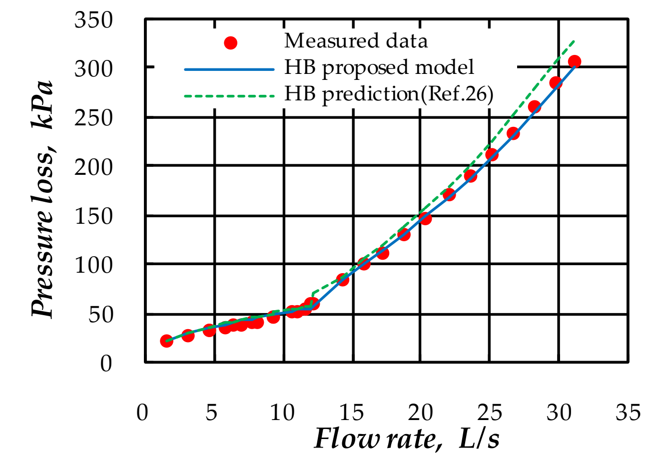


Fig.6. Comparison of predictions with measurements of Bentonite Mud 3 in rough pipe (HB model: τ_0=4.56957 Pa, K=1.54535 Pa·s^n, n=0.55037)

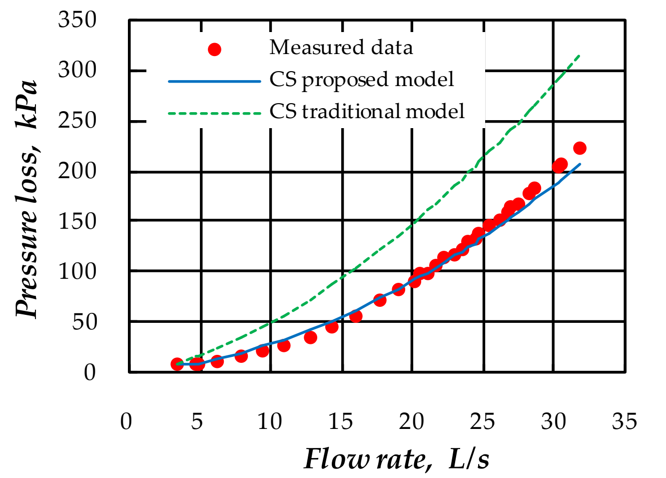


Fig.7. Comparison of predictions with measurements of MMH mud in smooth pipe (CS model: τ_0 = 3.4079 Pa, η_∞ = 0.00367 Pa·s)

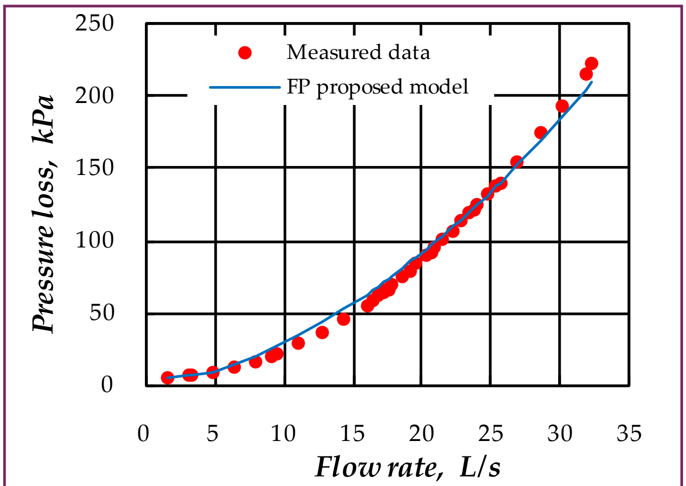


Fig.8. Comparison of predictions with measurements of Glycol mud in smooth pipe (FP model: τ_0 = 0.18209 Pa, a = 0.00472 Pa·s, b = 0.76365 Pa·s^c, c = 0.375)

Table 2

Detail hydraulic parameters of proposed model for system of figure 6

Q, L/s	Meas. ΔP, kPa	Calc. ΔP, kPa	Relative error, %	n'	D_{eff}, m	$\mu_{w,app}$, Pa·s	Re_g	f	$(Re_g)_{cr}$	Regime
1.56	21.60	22.21	2.84	0.41386	0.04615	0.2578	96	0.1674	2903	Laminar
3.19	28.97	30.26	4.44	0.44947	0.04784	0.1781	293	0.0546	2854	Laminar
4.58	33.25	35.70	7.36	0.46462	0.04851	0.1484	512	0.0312	2833	Laminar
4.69	32.96	36.10	9.52	0.46556	0.04855	0.1467	531	0.0301	2832	Laminar
5.75	36.27	39.69	9.44	0.47315	0.04888	0.1326	724	0.0221	2822	Laminar
6.39	39.48	41.76	5.78	0.47692	0.04904	0.1258	852	0.0188	2817	Laminar
6.91	39.29	43.34	10.30	0.47958	0.04915	0.1211	959	0.0167	2813	Laminar
7.77	41.07	45.85	11.65	0.48341	0.04931	0.1143	1146	0.0140	2808	Laminar
8.15	42.31	46.92	10.89	0.48492	0.04937	0.1117	1231	0.0130	2806	Laminar
9.34	46.25	50.15	8.45	0.48911	0.04955	0.1045	1515	0.0106	2800	Laminar
9.34	46.66	50.15	7.48	0.48910	0.04955	0.1045	1514	0.0106	2800	Laminar
10.66	51.40	53.50	4.08	0.49291	0.04970	0.0980	1847	0.0087	2795	Laminar
10.97	52.45	54.27	3.48	0.49372	0.04973	0.0967	1930	0.0083	2794	Laminar
11.58	56.24	55.73	0.90	0.49519	0.04979	0.0942	2093	0.0076	2792	Laminar
11.92	59.53	56.55	5.01	0.49598	0.04983	0.0929	2187	0.0073	2791	Laminar
12.26	61.14	57.35	6.20	0.49673	0.04986	0.0916	2282	0.0070	2789	Laminar
14.22	84.30	85.21	1.08	0.50053	0.05001	0.0853	2851	0.0077	2784	Turbulent
15.89	101.34	101.14	0.20	0.50321	0.05011	0.0809	3365	0.0074	2781	Turbulent
17.18	112.28	114.33	1.83	0.50502	0.05019	0.0779	3783	0.0071	2778	Turbulent
18.72	131.05	130.94	0.08	0.50693	0.05026	0.0748	4299	0.0069	2776	Turbulent
20.33	148.66	149.49	0.56	0.50871	0.05033	0.0719	4864	0.0066	2773	Turbulent
21.97	170.78	169.56	0.71	0.51032	0.05039	0.0693	5462	0.0065	2771	Turbulent
23.46	189.56	188.69	0.45	0.51163	0.05045	0.0672	6020	0.0063	2769	Turbulent
25.07	212.24	210.67	0.74	0.51292	0.05050	0.0651	6648	0.0062	2767	Turbulent
26.59	233.26	232.27	0.43	0.51402	0.05054	0.0634	7252	0.0060	2766	Turbulent
28.21	260.45	256.58	1.49	0.51511	0.05058	0.0616	7919	0.0059	2764	Turbulent
29.75	284.85	280.79	1.42	0.51606	0.05062	0.0601	8569	0.0058	2763	Turbulent
31.05	306.49	302.11	1.43	0.51681	0.05065	0.0589	9132	0.0058	2762	Turbulent

Table 2 gave the detailed data for various flow parameters that were calculated using the proposed model for the systems in figure 6. From the results, we can see that the fluid described is typical non-Newtonian fluid, with the n' values ranging from 0.4139 to 0.5168. Using the above regime transition criteria to determine the flow regime and utilizing the comparison of predicted to measured data, we can see there is an excellent match between the predictions and the experimental results with laminar flow, this is because the laminar flow pressure loss was obtained by the accurate flow equation. Also, the errors of laminar mainly depends on the accuracy that the selected rheological model which describe the non-Newtonian fluid rheological properties. For turbulent flow, the non-Newtonian pipe pressure losses calculation was unified based on the Re_g, D_{eff} and n'. Obviously, the calculated pressure losses also excellently match with the measured results. As shown in the tables, it also can be seen that as the flow rate increases, n' and D_{eff} increase as well.

4. Conclusions

The generalized flow index n' is assumed to be constant in traditional generalized flow index method for non-Newtonian fluid flow in pipes. However, the generalized flow index n' is non-constant for most non-Newtonian fluid except PL fluid due to their shear rate are variable. In the practical application, large errors will exist if this approximation hypothesis is made. A general expression of the generalized Reynolds number was derived which independent whether n' is constant. And the generalized flow index method would be extended to all time-independent non-Newtonian fluid pipe flow analysis. This method will provide a foundation for the popularization and application of some complex but more precise rheological models in engineering. The excellent application results show that it can precisely calculate pipe flow parameters over all the entire flow range for all types of different non-Newtonian flow.

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Обобщённая модель гидравлического расчета потока неньютоновской жидкости в трубе и оценка её применения

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Реферат

В статье рассматривается обобщенная гидравлическая модель потока неньютоновской жидкости в трубах. Обобщенная модель была разработана без учета того, что обобщенный показатель текучести (n') остается постоянным при всех скоростях сдвига. Основываясь на уравнении регулирования потока в трубе, были получены уравнения в явном виде, связывающие напряжение сдвига на стенке и объемный расход жидкости, полученные на основе различных реологических моделей, таких как Кэссона, Гершеля-Балкли и Робертсона-Стиффа, которые могут быть численно решены для получения достоверной скорости и напряжения сдвига на стенке. Приведен теоретический метод расчета для всех стационарных течений неньютоновских жидкостей. Получено выражение обобщенного числа Рейнольдса и представлен алгоритм вычислений падения давления.

Qeyri-nyuton mayenin boruda axınının ümumiləşdirilmiş hidravlik hesablama modeli və onun tətbiqinin qiymətləndirilməsi

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Xülasə

Məqalədə qetri-nyuton mayelərin boruda axınının ümumiləşdirilmiş hidravlik modeli təqdim olunur. Ümumiləşdirilmlş model ümumiləşdirilmiş axın göstəricisinin (n') sürüşmənin müftəlif sürətlərində sabit qalması nəzərə alınmadan işlənmişdir. Kesson, Qerşel-Balkli və Robertsin-Stif kimi reoloji modellər daha dəqiq divarboyu sürüşmə sürəti və sürüşmə təzyiqi almaq üçün rəqəmsal həll edilə bildiyi üçün məhz bu modellərin əsasında axının boruda tənzimlənməsi tənliyinə əsaslanaraq divarboyu sürüşmə təzyiqi və mayenin həcmi sərfi arasında dəqiq tənliklər alınmışdır. Qeyri-nyuton mayelərin bütün stasionar axınları üçün n' hesablamasının nəzəri üsulu göstərilir. Bundan əlavə ümumiləşdirilmiş Reynold rəqəminin ifadəsi əldə edilmişdir və təzyiq düşməsinin hesablanması alqoritmi təqdim olunur.