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NUMERICAL METHOD FOR STUDYING THE PROCESS OF MASS - HEAT TRANSFER IN DEFORMABLE LAYERS IN A CLASS OF DISCONTINUOUS FUNCTIONS

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A B S T R A C T

It is known that due to high thermobaric conditions during the operation of deep oil and gas fields, the parameters characterizing the rock and fluid change significantly depending on the effect of temperature and deformation. This, in turn, is reflected in the determination of factual index exploitations. The hydrodynamic models written taking into account the considered of changes form a system of nonlinear equations of the mixed type with specific characteristics and containing a high time derivative at the boundary. In numerous studies available in the literature, the solutions are found mainly using classical finite difference methods, without taking into account the characteristics of the mathematical problem. To ensure a reliable calculation of technological indicators, it is important to achieve that the hydrodynamic model adequately reflects of the process and find solutions that express all the properties of the physical process. The article proposes a solution to a mathematical model for studying the process of mass heat transfer in deformable deposits in a class of discontinues functions.

Keywords: mass-heat transfer; deformable collector; weak solution; finite differences in the class of truncated functions.

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Introduction

Assumed that a circular deformed layer of an oil with radius r_k and a height H is exploited by help a central well with radius r_c and with a production rate Q. Let us suppose that the reservoir temperature T_k, the contour and the well bottom pressure p_c and p_k change by the law $\bar{k}(p)=a_k[1+\alpha_k(p-p_s)]$, which is bed permeability [1]. Here, α_k and a_k are positive constants determined from field or laboratory data.

Under the following physical assumptions

- The well is complete according to its opening degree and character,
- Oil filtration obeys Darcy's law and it is non-isothermal,
- The pressure at the bed boundary is constant,
- The initial natural distribution of temperature along the layer is stable,
- The temperature of the fluid and porous medium are the same at any point in the layer,
- Heat transfer in the radial direction of the layer is negligible compared to the convective heat transfer,
- Influence of temperature changes in layer is not taken into account to the parameters defining of porous medium and fluid,
- The variation of the permeability parameter of the layer based on the deformation of the collector is expressed by a well-known empirical formula.

The considered problem is reduced to find of dynamic distributions of pressure and temperature functions in the reservoir from a dimensionless system of mixed differential equations with partial derivatives, under suitable initial and boundary conditions, [2-4]

$$\frac{\partial \bar{p}}{\partial \tau}=\frac{\partial}{\partial x}\left(x\frac{\bar{k}(\bar{p})}{\bar{\mu}}\frac{\partial \bar{p}}{\partial x}\right),\quad x_c\le x\le x_s,\quad \tau>0 \tag{1}$$

$$\frac{\partial \bar{T}}{\partial \tau}=\tilde{A}\frac{\bar{k}(\bar{p})}{\mu}x\frac{\partial \bar{p}}{\partial x}\left(\frac{\partial \bar{T}}{\partial x}+\tilde{B}\frac{\partial \bar{p}}{\partial x}\right)+\tilde{D}\frac{\partial \bar{p}}{\partial \tau},\quad x_c\le x\le x_s,\quad \tau>0 \tag{2}$$

$$\bar{p}(x.0)=p_0,\quad x_c\le x\le x_s \tag{3}$$

$$\frac{4\pi\bar{k}(\bar{p})H}{\bar{\mu}}\left(x\frac{\partial \bar{p}}{\partial x}\right)\Bigg|_{x=x_c}=\left(\tilde{Q}+\tilde{C}\frac{\partial \bar{p}}{\partial \tau}\right)\Bigg|_{x=x_c} \tag{4}$$

$$\bar{p}(r_s,\tau)=p_s,\quad \tau>0 \tag{5}$$

$$\bar{T}(x,\ 0)=T_0 \tag{6}$$

$$\bar{T}(x_k,\ 0)=T_s \tag{7}$$

Here,

$$x=\frac{\bar{r}^2}{4},\tau=\frac{k_0}{\mu_0 r_k^2\beta^*}t,\quad r_0=\frac{R_c}{R_k},\ 1=r_s,\ \frac{p_c}{p_k}=p_0,\ \frac{p_k}{p_k}=p_s,$$

$$T_0=\frac{T_c}{T_k},\quad T_s=\frac{T_k}{T_k},$$

$$\tilde{Q}=\frac{\mu_0}{Hk_0p_k}Q\ \text{ and }\ \tilde{C}=\frac{C}{Hr_k^2\beta^*},\quad \tilde{A}=\frac{\rho_f C_f p_k\beta^*}{C_{pl}},$$

$$\tilde{B}=\frac{p_k\varepsilon}{T_k},\quad \tilde{D}=\frac{\eta\rho_f C_f p_k}{C_{pl}T_k}$$

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and $x_c=\frac{r_0^2}{4}$. $x_s=\frac{r_s^2}{4}$, p_2, p_1, T_2 and T_1 represent pressure and temperature functions in the reservoir and well, respectively, β^* – elastic capacity coefficient of the reservoir [1/MPa], k – permeability coefficient [mkm^2], μ – dynamic viscosity [MPa·sec], r_c – radius of the well [m], r_k – radius of the layer [m], t – time variable [day], C_{pl} – volumetric heat capacity of the reservoir [Djol/(m^3·K)], C_{sr} – specific heat capacity of the rock [Djol/(kg·K)], C_f – specific heat capacity of the fluid [Djol/(kg·K)], P_f – density [kg/m^3], m – porosity, ε – Joule-Thomson coefficient [K/MPa], η – coefficient of adiabatic expansion [K/MPa], p_k – reservoir pressure [MPa], T_k – reservoir temperature [K], H – reservoir thickness [m], Q – well debit [m^3/sec], C – borehole influence coefficient [m^3/MPa], P_1 – well pressure [MPa], T_1 – well temperature K°, v – fluid velocity in the well [m/sec], ψ – coefficient of hydraulic resistance, L – length of the wellbore [m], α – thermal conductivity of the wellbore KKal/(L^2·T·C^0) , γ – specific gravity of the fluid, w – fluid percolation rate in the reservoir [m/sec], g – free-fall acceleration [m/sec^2], t_{exp} – working life of the well [day], G – geothermal gradient [K°/M], L_1 and L_2 show the coordinates of the formation bottom, ceiling and the device placed in the well [M].

Later the problem (1)-(7) we will call as the main problem.

Numerical solution to the problem

As can be seen, (1) and (2) are non-linear partial differential equations. As is known from the literature, in the processes expressed by equation (1), the front of the excited region spreads over a finite distance during a finite time, that is, $\sup\limits_x p(x,\ t)=\{x_c<x\le x_f(t),\ t>0\}$, [5, 6], in order words, the pressure have the nature of localization. The localization effect presents in the pressure function affects also to dynamics the of temperature field. In other words, the temperature front extends over a finite distance in a finite time. Having of these singularity violate of smoothness of the pressure and temperature functions. Hence, the classical solutions of the considered problem may be not exist.

The results obtained by the applied finite difference method without taking into account these properties of the solutions of equation (1), (2) may be not express of the physical process correctly. In order to overcome this difficulty, an auxiliary problem, which has advantages not in the main problem, is introduced [7, 8].

Auxiliary problem

Since the solution of the nonlinear parabolic type equation has a localization effect, the classical solution of the equations (1) and (2) may be not exist [5, 6]. From stated It is clear that the properties the order of differentiability of function $\overline{p}\,(x, t)$ is one unit less than the order of differentiatiablity required of equation (1) from the solution. This does not allow to solve the problem (1)-(7) by finite difference method. To find of weak solution of the problem, instead system of auxiliary equations is proposed for the main problem [8]

$$\frac{\partial}{\partial\tau}\int_{x_c}^{x}\overline{p}\left(\xi,\tau\right)d\xi=$$
$$=x\frac{\overline{k}\left(\overline{p}\right)}{\overline{\mu}}\frac{\partial\overline{p}\left(x,\tau\right)}{\partial x}-\frac{1}{4\pi\mathrm{H}}\left[\tilde{Q}+\tilde{C}\frac{\partial\overline{p}\left(x,\tau\right)}{\partial\tau}\bigg|_{x=x_c}\right] \tag{8}$$

$$\frac{\partial}{\partial\tau}\int_{x_c}^{x}\left[\overline{T}\left(\xi,\tau\right)-\tilde{D}\overline{p}\left(\xi,\tau\right)\right]d\xi=$$
$$=x\frac{\overline{k}\left(\overline{p}\right)}{\overline{\mu}}\frac{\partial\overline{p}\left(x,\tau\right)}{\partial x}-\frac{1}{4\pi\mathrm{H}}\left[\tilde{Q}+\tilde{C}\frac{\partial\overline{p}\left(x,\tau\right)}{\partial\tau}\bigg|_{x=x_c}\right]- \tag{9}$$
$$-\int_{x_c}^{x}\left[\overline{T}\left(\xi,\tau\right)+\tilde{B}\overline{p}\left(\xi,\tau\right)\right]\frac{\partial\overline{p}\left(\xi,\tau\right)}{\partial\tau}d\xi$$

It is required to find the solution of the system of equations (8), (9) within the initial and boundary conditions (3) - (7). We will call problem (8), (9), (3) - (7) auxiliary issues. The auxiliary problem has the following advantages:

- since the $w(x,t)=x\frac{\overline{k}\left(\overline{p}\right)}{\overline{\mu}}\frac{\partial\overline{p}\left(x,t\right)}{\partial x}$ lying in the right part of equations of (8), (9) physically represents of the flow is continuous function,
- the function $\overline{p}\left(x,\tau\right)$ is continuous, and the first derivative of $\overline{p}\left(x,\tau\right)$ with respecr to x function of second type discontinuous that is, the function $\frac{\partial\overline{p}\left(x,\tau\right)}{\partial x}$ does not exist in the large [5, 6].

Now, we establish the following grids on the intervals $[x_c, x_s]$ and [0,T)

$$\omega_{h_x,\tau}=\left\{x_j=x_c+jh_x,\ \ j=0,1,\ldots,n;\ \tau_k=kh_\tau,\ \ k=0,1,2,\ldots\right\}$$
$$\omega_{h_z,\tau}=\left\{z_\nu=0+\nu h_z,\ \ \nu=0,1,\ldots,m;\ \tau_k=kh_\tau,\ \ k=0,1,2,\ldots\right\}$$

The finite differences schema for integro-differential equations (8), (9) are written as follows:

$$\frac{\partial}{\partial\tau}\sum_{j=1}^{i}\int_{x_{j-1}}^{x_j}\overline{p}\left(\xi,\tau\right)d\xi=x_i\frac{k\left(\overline{p}\left(x_i,\tau_k\right)\right)}{\overline{\mu}}\frac{\left(\mathcal{P}_{i,k}-\mathcal{P}_{i-1,k}\right)}{h}-$$
$$-\frac{x_c\overline{\mu}}{4\pi\mathrm{H}}\left[\tilde{Q}+\tilde{C}\frac{\left(\mathcal{P}_{0,k+1}-\mathcal{P}_{0,k}\right)}{h_\tau}\right] \tag{10}$$

$$\frac{\partial}{\partial\tau}\sum_{j=1}^{i}\int_{x_{j-1}}^{x_j}\left[\overline{T}\left(\xi,\tau\right)-\tilde{D}\overline{p}\left(\xi,\tau\right)\right]d\xi=$$
$$=x_i\frac{k\left(\overline{p}\left(x_i,\tau_k\right)\right)}{\overline{\mu}}\frac{\left(\mathcal{P}_{i,k}-\mathcal{P}_{i-1,k}\right)}{h}-\frac{1}{4\pi\mathrm{H}}\left[\tilde{Q}+\tilde{C}\frac{\left(\mathcal{P}_{0,k+1}-\mathcal{P}_{0,k}\right)}{h_\tau}\right]-$$
$$-\sum_{j=1}^{i}\int_{x_{j-1}}^{x_j}\left[\overline{T}\left(\xi,\tau\right)+\tilde{B}\overline{p}\left(\xi,\tau\right)\right]\frac{\left(\mathcal{P}_{j,k+1}-\mathcal{P}_{j,k}\right)}{h_\tau}d\xi, \tag{11}$$
$$(i=1,\ 2,\ \ldots,\ n;\ \ k=0,\ 1,\ 2,\ \ldots,)$$

Appling the method of rectangles to the integrals included in the system of equations (10), (11), for solving the main problem we get the following algorithm:

$$\mathcal{P}_{i,k+1}=\mathcal{P}_{i,k}-\sum_{j=1}^{i-1}\left(\mathcal{P}_{j,k+1}-\mathcal{P}_{j,k}\right)+\frac{1}{\overline{\mu}}\frac{h_\tau}{h_x^2}x_i\,k\left(\mathcal{P}_{i,k}\right)\left(\mathcal{P}_{i,k}-\mathcal{P}_{i-1,k}\right)-$$
$$-\frac{x_c\overline{\mu}\,h_\tau}{4\pi\mathrm{H}h_x}\left[\tilde{Q}+\tilde{C}\frac{\left(\mathcal{P}_{0,k+1}-\mathcal{P}_{0,k}\right)}{h_\tau}\right] \tag{12}$$

$$T_{i,k+1}=T_{i,k}+\tilde{D}\left(\mathcal{P}_{i,k+1}-\mathcal{P}_{i,k}\right)-\sum_{j=1}^{i-1}\left[\left(T_{j,k+1}-T_{j,k}\right)-\tilde{D}\left(\mathcal{P}_{j,k+1}-\mathcal{P}_{j,k}\right)\right]$$
$$+x_i\frac{h_\tau}{h_x^2}\frac{k(\overline{p}\left(x_i,\tau_k\right)}{\overline{\mu}}\left(\mathcal{P}_{i,k}-\mathcal{P}_{i-1,k}\right)-\frac{h_\tau}{4\pi\mathrm{H}h_x}\left[\tilde{Q}+\tilde{C}\frac{\left(\mathcal{P}_{0,k+1}-\mathcal{P}_{0,k}\right)}{h_\tau}\right]- \tag{13}$$
$$-\sum_{j=1}^{i}\left(\left(T_{j,k}+\tilde{B}\mathcal{P}_{j,k}\right)\left(\mathcal{P}_{j,k+1}-\mathcal{P}_{j,k}\right)\right),$$
$$\left(i=1,2,\ldots,n-1;k=0,1,2,\ldots\right)$$

The initial and boundary conditions for (12)- (13) will be

$$\mathcal{P}\left(x_i,0\right)=p_s,\quad i=0,1,2,\ldots,n, \tag{14}$$

$$\mathcal{P}\left(x_n,\tau_k\right)=p_s,\quad k>0 \tag{15}$$

$$T_{i,0}=1,\quad \left(i=0,1,2,\ldots,n\right) \tag{16}$$

$$T_{0,k}=1,\quad \left(k=0,1,2,\ldots,\right) \tag{17}$$

As you can see, problem (12)-(17) form a two-point system, that is, in each equation of the system there are only two unknowns, which makes it easy to find their solutions. The auxiliary problem makes it possible to prove the convergence and stability of difference schemes, and also provides simple and efficient computer programming. The solutions obtained by the proposed algorithms correctly reflect all the properties of the physical process.

Conclusion

Taking into account thermobaric effects, a new finite difference method for finding a numerical solution to the system of nonlinear differential equations describing the fluid flow in a deforming formation and under suitable initial and boundary conditions is proposed. The proposed algorithm makes it possible to study of the dynamic changes of temperature and pressure functions at any point and at any time in the reservoir.

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Численный метод исследования процесса массо-теплообмена в деформируемых слоях в классе разрывных функций

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Реферат

Известно, что в связи с высокими термобарическими условиями при эксплуатации глубокозалегающих месторождений нефти и газа параметры, характеризующие породу и флюид существенно изменяются в зависимости от влияния температуры и деформации. При фундаментальных исследованиях по физико-математическому моделированию важно учитывать все особенности температурных аномалий при движении газожидкостных систем на месторождениях углеводородов с различными термобарическими условиями при определении прогноза полной разработки, чему и посвящена настоящая работа. В статье решается задача о влиянии температурных аномалий на распределение термогидродинамических полей при разработке нефтяной залежи деформируемым пластом. Предложен алгоритм численного решения и проведены многочисленные компьютерные эксперименты путем создания программного обеспечения. Оценивалось влияние удельной теплоемкости жидкости, коэффициента Джоула-Томсона, коэффициента влияния объема ствола скважины, абсолютной проницаемости, вязкости жидкости и деформационных свойств на процесс.

Ключевые слова: массо-теплоперенос; деформируемый коллектор; слабое решение; конечные разности в классе усеченных функций.

Deformasiya olunan yataqlarda kütlə-istilik mübadiləsi prosesinin kəsilən funksiyalar sinfində ədədi həll üsulu

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Xülasə

Karbohidrogen ehtiyatlarının mövcudluğunu təmin edən çoxsaylı səbəblər, həmçinin onların əmələ gəlmə və çıxarılması ilə bağlı baş verən proseslərin mürəkkəbliyi işlənilmə üsullarının rasional olaraq seçilməsi və yaradılmasında kompleks elmi yanaşmaların zəruriliyini önə çəkir. Bu tədqiqatların ən vacib istiqaməti məsaməli mühitdə çoxfazlı fluidin süzülməsi şəraitində layda təzyiq və temperatur dəyişmələrini nəzərə almaqla karbohidrogenlərin kütlə mübadiləsini və fazalararası keçid mexanizmlərinin öyrənilməsindən ibarətdir. Fiziki və riyazi modelləşmə üzrə aparılan fundamental tədqiqatlar ilə müxtəlif səciyyəli termobarik səraitə malik karbohidrogen yataqlarında qaz-maye sistemlərinin hərəkəti zamanı yaranan temperatur anomaliyalarının bütün xüsusiyyətlərinin tamliqla işlənilmə göstəricilərinin proqnozunun təyini zamanı nəzərə alınması vacib və mühüm əhəmiyyətə malikdir. Məqalə bu qəbildən olan işlənilmə məsələsinin həllinə həsr olunduğundan mühüm aktuallıq kəsb edir. Məqalədə yuxarıda qeyd edilənləri nəzərə almaqla, sadəlik üçün, kollektoru deformasiya olunan neft layının işlənilməsi prosesində yaranan temperatur anomaliyalarının termohidrodinamiki sahələrin paylanmasına təsiri prossesinin hidrodinamik modeli öyrənilmişdir. Prossesin fiziki xassələrini düzgün əks etdirən ədədi həll alqoritmi təklif edilmiş və proqram təminatı yaradılaraq çoxsaylı riyazi eksperimentlər aparılmışdır.

Açar sözlər: kütlə-istilik ötürmə; deformasiya olunan kollektor; zəif həll; kəsilmiş funksiyalar sinfində sonlu fərqlər.